The appearance, motion, and disappearance of three-dimensional magnetic null points

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While theoretical models and simulations of magnetic reconnection often assume symmetry such that the magnetic null point (when present) is co-located with a flow stagnation point, the introduction of asymmetry typically leads to non-ideal flows across the null point. To understand this behavior, we derive exact expressions for the motion of three-dimensional linear null points. Our most general expression shows that linear null points move in the direction along which the vector field and its time derivative are antiparallel. For resistive magnetohydrodynamics, null-point motion results from advection by the bulk plasma flow and resistive diffusion of the magnetic field. Null-point motion is described intrinsically by parameters evaluated locally; however, global dynamics help set the local conditions at the null. Null points appear and disappear through non-ideal effects such as resistive diffusion, which sets additional constraints on bifurcation analyses. The instantaneous velocity of separation or convergence of a bifurcating null-null pair will typically be infinite, but may be finite or zero when certain conditions are met. However, the motion of separators cannot be described using solely local parameters because the identification of a particular field line as a separator may change as a result of non-ideal behavior elsewhere along the field line.

Introduction.—Magnetic reconnection [e.g., 1, and references therein] frequently occurs at and around magnetic null points: locations where the magnetic field strength equals zero [2–6]. Magnetospheric null points have been identified using multipoint *in situ* measurements as the nulls pass through the spacecraft constellation [7]. Null points in the solar atmosphere have been identified through extrapolation of the photospheric magnetic field and morphology in coronal emission [8].

Two-dimensional, non-degenerate magnetic null points are classified as X-type or O-type depending on the local magnetic field structure. If we define $\mathbf{M}$ as the Jacobian matrix of the magnetic field at the null point, then a null point will be X-type if $\det \mathbf{M} < 0$, O-type if $\det \mathbf{M} > 0$, and degenerate if $\det \mathbf{M} = 0$. Magnetic reconnection in two dimensions can only occur at null points [e.g., 9, 10]. In three dimensions, however, the structure of magnetic null points is significantly more complex [2–6]. Null lines and null planes are structurally unstable and unlikely to exist except as a transition between topological states [e.g., 4, 11]. The magnetic field structure around a linear three-dimensional null point includes separatrix surfaces of infinitely many field lines that originate (or terminate) at the null, and two spine field lines that end (or begin) at the null. Separators are magnetic field lines that connect two nulls. Null points, separatrix surfaces, spines, and separators are the topological boundaries that divide the magnetic field into distinct domains and are therefore preferred locations for magnetic reconnection [12]. Three-dimensional magnetic reconnection can also occur without nulls [10, 13], especially in regions where the magnetic connectivity changes quickly.

Motion of magnetic null points and reconnection regions occurs during any realistic occurrence of magnetic reconnection. Magnetotail X-lines frequently retreat in the tailward direction [14]. At the dayside magnetopause [15] and in tokamaks [16], the combination of a plasma-pressure gradient and a guide field leads to diamagnetic drifting of the reconnection site that can suppress reconnection. Laboratory experiments frequently show reconnection site motion, often due to geometry or the Hall effect [17–19]. During solar flares, the reconnection site often rises with time as the flare loops grow and can also show transverse motions [e.g., 20].

Theoretical models of magnetic reconnection often assume symmetry such that each magnetic null coincides with a flow stagnation point in the reference frame of the system. When asymmetry is introduced, there is in general a separation between these two points [18, 21–24], and in some cases a stagnation point might not even exist near a null point [25]. In all of these situations, there will, in general, be plasma flow across the magnetic null and the null will change position. Interestingly, the velocity of a null point will, in general, not equal the plasma flow velocity at the null point [23–25]. This is similar to the flow-through mode of reconnection [26].

In previous work [23], an exact expression was derived for the motion of an X-line when its location is constrained to one dimension by symmetry. In resistive magnetohydrodynamics (MHD), contributions to X-line motion come from a combination of bulk plasma flow and resistive diffusion of the normal component of the magnetic field. In this Letter, we derive exact expressions for the motion of null points in multiple dimensions.

Motion of Linear Null Points in an Arbitrary Vector Field.—We define $\mathbf{x}_n(t)$ as the time-dependent position of an isolated null point in a vector field $\mathbf{B}$. We define $\mathbf{B}_n\left(\mathbf{x}_n(t), t\right)$ as the value of the vector field at the null; while $\mathbf{B}_n \equiv 0$ for all time, $\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n(t)} \equiv \frac{\partial \mathbf{B}_n}{\partial t} \neq 0$ when the null point is moving. We define $\mathbf{U}$ to be the velocity of this null,

$$\mathbf{U} \equiv \frac{\mathrm{d}\mathbf{x}_n}{\mathrm{d}t}. \tag{1}$$

The local structure of a non-degenerate null point can be found by taking a Taylor expansion and keeping the linear terms [2–6]. The linear structure is then given by

$$\mathbf{B} = \mathbf{M}\mathbf{r}, \tag{2}$$

where $\mathbf{r} \equiv \mathbf{x} - \mathbf{x}_n$. The elements of the Jacobian matrix $\mathbf{M}$ evaluated at the null are given by

$$M_{ij} = \partial_j B_i, \tag{3}$$

where i is the row index and j is the column index. The trace of $\mathbf{M}$ equals zero for $\nabla \cdot \mathbf{B} = 0$.

Next we take the derivative following the motion of the null,

$$\frac{\partial \mathbf{B}_n}{\partial t} + \left(\mathbf{U} \cdot \nabla\right) \mathbf{B}\big|_{\mathbf{x}_n} = 0. \tag{4}$$

This expression gives the total derivative of the magnetic field at the null point using the null's velocity in an arbitrary reference frame. This derivative equals zero because the magnetic field at the null by definition does not deviate from zero as we are following it. By solving for $\mathbf{U}$ in Eq. 4, we arrive at our most general expression for the velocity of the null point,

$$\mathbf{U} = -\mathbf{M}^{-1} \frac{\partial \mathbf{B}_n}{\partial t}, \tag{5}$$

which is valid for vector fields of arbitrary dimension.

An alternate derivation for Eq. 5 starts from the equation for the time evolution of $\mathbf{B}$ near a linear null point,

$$\mathbf{B}\left(\mathbf{x}, \delta t\right) = \mathbf{M}\mathbf{x} + \frac{\partial \mathbf{B}_n}{\partial t} \delta t, \tag{6}$$

where we assume the null point is at the origin at $t = 0$. This expression is valid when δt and $|\mathbf{x}|$ are small. We define $\delta \mathbf{x}_n$ as the position of the null point at δt. Setting $\mathbf{B}\left(\delta \mathbf{x}_n, \delta t\right) = 0$ provides a unique solution for $\mathbf{U} \equiv \delta \mathbf{x}_n / \delta t$, and we again arrive at Eq. 5.

Equation 5 shows that a null point will move along the path for which $\mathbf{B}$ and $\frac{\partial \mathbf{B}_n}{\partial t}$ are oppositely directed. The null point will move faster if the vector field is changing quickly in time or varying slowly in space along this path. This exact result for $\mathbf{U}$ can be applied to find the velocity of linear null points in any time-varying vector field with continuous first derivatives in time and space about the null point. A unique velocity $\mathbf{U}$ exists as long as $\mathbf{M}$ is non-singular. If $\mathbf{M}$ is non-singular, then there exists exactly one radial path away from the null for which the vector field is pointed in a particular direction.

Motion of Magnetic Null Points.—We now consider the case where $\mathbf{B}$ is a magnetic field. The derivation of Eq. 5 does not invoke any of Maxwell's equations. We now introduce Faraday's law,

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{7}$$

where $\mathbf{E}$ is the electric field. By combining Eqs. 5 and 7, we arrive at the relation

$$\mathbf{U} = \mathbf{M}^{-1} \left(\nabla \times \mathbf{E}\right), \tag{8}$$

which additionally requires continuous first derivatives of the electric field in space about the null point. This expression does not depend on any particular Ohm's law, and indeed can be applied in situations where there is no Ohm's law.

Next we consider the resistive MHD Ohm's law,

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J}, \tag{9}$$

where $\mathbf{V}$ is the plasma flow velocity and $\mathbf{J}$ is the current density. The resistivity η is assumed to be uniform for simplicity. Eq. 8 then becomes

$$\mathbf{U} = \mathbf{V} - \eta \mathbf{M}^{-1} \nabla^2 \mathbf{B}, \tag{10}$$

where all quantities on the right hand side are evaluated at the magnetic null. This expression requires that $\mathbf{B}$ have continuous first derivatives in time and continuous second derivatives in space about the null point. As shown in Ref. 23, null point motion in resistive MHD results from a combination of advection by the bulk plasma flow and resistive diffusion of the magnetic field. Even in the absence of flow, null points may still move in resistive situations.

Equation 8 can also be evaluated using an Ohm's law containing additional terms. For example, we can choose our Ohm's law to be

$$\mathbf{E} + \mathbf{V}_i \times \mathbf{B} = \eta \mathbf{J} + \frac{\mathbf{J} \times \mathbf{B}}{e n_e} - \frac{\nabla p_e}{e n_e}, \tag{11}$$

where $\mathbf{V}_i$ is the bulk ion velocity, n_e is the electron density, e is the elementary charge, and p_e is a scalar electron pressure. For $\mathbf{J} = e n_e \left(\mathbf{V}_i - \mathbf{V}_e\right)$, Eq. 8 becomes

$$\mathbf{U} = \mathbf{V}_e - \eta\, \mathbf{M}^{-1} \nabla^2 \mathbf{B} + \mathbf{M}^{-1} \left(\frac{\nabla n_e \times \nabla p_e}{n_e^2 e} \right) \tag{12}$$

where quantities are again evaluated at the null point. The first term on the right hand side corresponds to the magnetic field being carried with the electron flow velocity, $\mathbf{V}_e$, rather than the bulk plasma flow, the second term corresponds to the resistive diffusion of the magnetic field at the null, and the third term corresponds to the Biermann battery.

Just as we must be careful in describing the motion of magnetic field lines [27], we must also be careful in describing the motion of magnetic null points. Null points are not objects. A null point is not permanently affixed to a parcel of plasma except in ideal or perfectly symmetric cases. Null points cannot be pushed directly by plasma pressure gradients or other forces on the plasma, but there will, in general, be indirect coupling between the momentum equation and Faraday's law that contributes to null-point motion. The motion of a null point is determined intrinsically by local quantities evaluated at the null point. However, global dynamics help set the local conditions that determine null point motion.

In addition to providing insight into the physics of non-ideal flows at magnetic null points and constraining models of asymmetric reconnection, these expressions have several practical applications. Locating nulls of vector fields in three dimensions is non-trivial [28], but these expressions provide initial guesses for the positions of null points at future times. When there are several null points clustered near each other, these expressions provide a method for identifying which null points correspond to each other at different times. These expressions provide a test of numerical convergence and can be used to estimate the effective numerical resistivity in simulations. A practical limitation is that these expressions will often require taking derivatives of numerical or noisy data.

The appearance and disappearance of magnetic null points.—We next consider the implications of this analysis on the emergence and disappearance of magnetic null points. The local approach taken in this Letter complements global-bifurcation studies [29]. Thus far we have only considered non-degenerate null points for which the local magnetic field can be described by Eq. 2 using only the linear terms in the Taylor series expansion. As long as $\mathbf{M}$ is non-singular, then there exists a unique velocity corresponding to the motion of that null point. Non-degenerate null points are therefore structurally stable and cannot disappear unless $\mathbf{M}$ becomes singular.

Non-degenerate null points are structurally unstable and generally exist instantaneously as a transition between different topological states. To describe the bifurcation of an isolated degenerate null point self-consistently, the linear representation in Eq. 2 is insufficient and we must include higher-order terms in the Taylor expansion of the magnetic field. Degenerate null points will bifurcate into multiple nulls along the radially outward paths on which $\mathbf{B}$ and $\frac{\partial \mathbf{B}_n}{\partial t}$ are antiparallel. If no such directions exist, then the degenerate null point disappears.

The velocity of convergence or separation of a null-null pair at the instant of bifurcation may be found by taking the limit of Eq. 5. The denominator for all components of velocity approaches zero because $\det \mathbf{M} = 0$ at the instant of bifurcation. However, if the numerator equals zero for all components of velocity (e.g., if $\frac{\partial \mathbf{B}_n}{\partial t} = 0$), then the instantaneous velocity may be infinite, finite, or zero. In analytic cases, this velocity may be found by using l'Hôpital's rule. As an example of a time-dependent magnetic field which undergoes a bifurcation at the origin at $t = 0$, consider a magnetic field configuration given by

$$\mathbf{B}\left(\mathbf{x}, t\right) = \left(\begin{array}{c} (a-z)x + by \\ cx - (a+z)y \\ z^2 - \mathrm{sgn}\left(t\right) \left|t\right|^{\alpha} \end{array} \right), \tag{13}$$

where $\alpha > 0$ and a, b, and c are arbitrary constants with $a^2 + bc \neq 0$. For $t > 0$, the third component of $\mathbf{B}$ reduces to $z^2 - t^{\alpha}$ and two null points exist, but when $t < 0$, there are no null points. At $t = 0$, a single second-order null point appears at the origin as the system undergoes a saddle-node bifurcation. For $t > 0$, the two null points are at $\mathbf{x}_n = \left(0, 0, \pm t^{\alpha/2}\right)$ and have velocities of $\dot{\mathbf{x}}_n = \left(0, 0, \pm \frac{\alpha}{2} t^{\alpha/2-1}\right)$. When $0 < \alpha < 2$, the velocity of separation diverges to infinity at $t = 0$. For the critical case when $\alpha = 2$, the null-point velocities are constant: $\dot{\mathbf{x}}_n = (0, 0, \pm 1)$. When $\alpha > 2$, the null-point velocities asymptotically approach zero at $t = 0$. Under typical circumstances, the instantaneous velocities of separation or convergence for a bifurcating null-null pair will be infinite. However, some situations may arise when certain symmetries are present that yield instantaneous velocities that are finite or zero.

In resistive MHD, null points must diffuse in and out of existence. Topological analyses show that linear magnetic nulls must emerge and disappear in pairs [29]. In the reference frame of the moving plasma, a necessary condition for a degenerate null point to form is that the resistive term in the induction equation, $\eta \nabla^2 \mathbf{B}$, be antiparallel to the magnetic field at the location of the impending degenerate null. This places geometric constraints on which bifurcations are allowed to happen.

The motion of separators.—Finally, we may ask whether or not a similar local analysis can be performed to describe the motion of separators. Consider a separator connecting two nulls. Suppose also that a segment of the separator exhibits non-ideal evolution, while the rest of the separator evolves ideally. At a slightly later time, the field line that was the separator will, in general, not continue to be the separator between these two nulls despite the locally ideal evolution. Therefore, the motion of separators cannot be described using solely local parameters. However, it may be possible to derive an expression for the motion of a separator by taking into account plasma flow and connectivity changes along its entire length. Such an approach would provide insight into the structural stability of separators and separator bifurcations (see also Ref. 30).

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