Statistical and spectral properties of magnetic islands in reconnecting current sheets during two-ribbon flares

Chengcai Shen, Jun Lin, Nicholas A. Murphy, and John C. Raymond

Citation: Phys. Plasmas 20, 072114 (2013); doi: 10.1063/1.4816711
View online: http://dx.doi.org/10.1063/1.4816711
View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v20/i7
Published by the AIP Publishing LLC.

Additional information on Phys. Plasmas
Journal Homepage: http://pop.aip.org/
Journal Information: http://pop.aip.org/about/about_the_journal
Top downloads: http://pop.aip.org/features/most_downloaded
Information for Authors: http://pop.aip.org/authors
Statistical and spectral properties of magnetic islands in reconnecting current sheets during two-ribbon flares

Chengcai Shen,1,2,3 Jun Lin,1,2 Nicholas A. Murphy,2 and John C. Raymond2
1Yunnan Astronomical Observatory, Chinese Academy of Sciences, P.O. Box 110, Kunming, Yunnan 650011, China
2Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138, USA
3Graduate School of the Chinese Academy of Sciences, Beijing 100039, China

(Received 22 March 2013; accepted 24 June 2013; published online 30 July 2013)

We perform a set of two dimensional resistive magnetohydrodynamic simulations to study the reconnection process occurring in current sheets that develop during solar eruptions. Reconnection commences gradually and produces small-scale structures inside the current sheet, which has one end anchored to the bottom boundary and the other end open. The main features we study include plasmoids (or plasma blobs) flowing in the sheet, and X-points between pairs of adjacent islands. The statistical properties of the fine structure and the dependence of the spectral energy on these properties are examined. The flux and size distribution functions of plasmoids roughly follow inverse square power laws at large scales. The mass distribution function is steep at large scales and shallow at small scales. The size distribution also shows that plasmoids are highly asymmetric soon after being formed, while older plasmoids tend to be more circular. The spectral profiles of magnetic and kinetic energy inside the current sheet are both consistent with a power law. The corresponding spectral indices $\gamma$ are found to vary with the magnetic Reynolds number $R_m$ of the system, but tend to approach a constant for large $R_m (> 10^5)$. The motion and growth of blobs change the spectral index. The growth of new islands causes the power spectrum to steepen, but it becomes shallower when old and large plasmoids leave the computational domain. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4816711]

I. INTRODUCTION

The reconnecting current sheet is a common structure that forms above two-ribbon flares.1 Theoretical studies predict that it extends from the top of flare loops to the associated coronal mass ejection (CME).2–4 In observations, several eruptive events have been reported to develop long current sheets observed by various instruments, including TRACE, Hinode/XRT, SOHO/UVCS, and SDO/AIA.5–10 Plasmoids or plasma blobs of different sizes were found to flow along the current sheet either toward or away from the sun.5–8,11–18

In the eruptive process, the closed magnetic configuration is stretched severely, and a long sheet develops and becomes unstable to several magnetohydrodynamic (MHD) instabilities. Among these instabilities, the tearing mode is the most important and has been extensively studied. It was investigated for the first time by Furth et al.,19 who showed that it occurs on a time scale $\tau$ such that $\tau_A < \tau < \tau_d$. Using the characteristic length of system $l$ and the resistivity $\eta$, $\tau_d = l^2/\eta$ and $\tau_A = l/v_A$ are the times it takes to traverse the sheet at the diffusion speed $v_d = \eta/l$, and at the Alfvén speed $v_A$, respectively. The tearing mode causes the current sheet to form a chain of plasmoids or magnetic islands (see figure 6.3 of Priest and Forbes).20 Loureiro et al.21 showed that a Sweet-Parker current sheet with a large aspect ratio is unstable to long wave length perturbations in an environment of high Lundquist (or magnetic Reynolds) number $R_m$. The maximum growth rate of the instability in the linear stage scales as $R_m^{1/4}v_A/L_{cx}$, where $L_{cx}$ is the length of the current sheet.

The formation and evolution of plasma blobs in current sheets of two-ribbon flares have been numerically studied for more than two decades. The results of Forbes and Malherbe22 and Riley et al.23 suggested that the tearing mode instability plays an important role in magnetic field diffusion and in governing the scale of the sheet. Báta et al.24 reported a broad variety of kinematic/dynamic properties of plasmoids. Recently, similar processes related to plasmoids inside extended current sheets have been studied extensively in the plasma physics community as well. Resistive MHD simulations have confirmed and further investigated the scaling of this instability, showing that formation of plasmoids leads to a reconnection rate much faster than that predicted by the Sweet-Parker model.25–32 However, Shepherd and Cassak33 argued that collisionless effects, which become important on scales comparable to the ion inertial length, are needed to explain the observed reconnection rates. Secondary island formation is also observed in particle-in-cell (PIC) simulations of elongated current sheets.34

Statistical descriptions for magnetic islands formed in current layers have recently been developed. Fermo et al.35,36 introduced the distribution function ($f$) using two parameters to describe magnetic islands, which are the flux contained in the island and the area it encloses. This model shows that the distribution function is determined by the generation of secondary islands, the growth rate of islands, and their merging. Their numerical solution also indicates that the distribution function decays exponentially with the flux. Uzdensky et al.37 reported that the self-similar distribution...
function of plasmoid size and fluxes follows an inverse square law, which is generally confirmed by Loureiro et al.\textsuperscript{37} in numerical simulations. Loureiro et al.\textsuperscript{37} reported that the plasmoid flux and half-width distribution functions scale as an inverse square law. They suggested that plasmoids populate a triangular region $w_{c} \geq \Psi/B_{0}$ because of plasmoid coalescence. Recently, Huang and Bhattacharjee\textsuperscript{38} developed a phenomenological analytic model and numerical simulations. By considering the velocity of magnetic islands in the distribution function, they reported that the distribution function follows a power law with $f(\psi) \sim \psi^{-1}$.

On the other hand, by performing spectral analyses on regions of magnetic reconnection, a large number of studies have reported on the relationship between turbulence and instabilities during reconnection. Matthaeus and Lamkin\textsuperscript{39} studied the detailed properties of turbulent reconnection in the nonlinear phase by using spectral methods. They investigated the 1D and 2D spectral properties of turbulent reconnection on a set of numerical MHD simulations. Furthermore, the statistical properties of multiscale reconnection events have been reported in simulations of 2D turbulent reconnection.\textsuperscript{40-42} Bára et al.\textsuperscript{43} point out that the cascading process naturally occurs inside a fragmented current sheet displaying a power-law distribution of the magnetic field energy in 2D MHD simulations, and that the dissipation scale is a few times $10^{2}$ km. They suggest that the cascading reconnection and fragmenting and coalescence processes play important roles for the magnetic energy transfer to smaller scale during the reconnection process.

In this work, we focus on the statistical and spectral properties of magnetic islands or plasmoids in reconnecting current sheets in MHD simulations of a two-ribbon flare. The formation of plasmoids changes the distribution of magnetic energy on different scales. During the formation of plasmoids, magnetic energy develops on small scales. Due to the complicated dynamics of plasmoids, such as the growth of a single plasmoid or the merging of two plasmoids, magnetic energy is transferred from small scales to intermediate scales. We perform a Fourier analysis to investigate the evolution of magnetic islands and study the distribution of magnetic energy on different scales.

This paper is organized as follows. In Sec. II, we describe the methods of 2D MHD simulations. In Sec. III, we discuss the critical magnetic Reynolds number ($R_{m}$) for the onset of the tearing of the current sheet, and show how the critical $R_{m}$ is impacted by initial perturbation and line-tied boundary conditions. In Sec. IV, we describe the magnetic flux, size, and mass distribution functions. In Secs. V, VI, and VII, we study the magnetic and kinetic energy spectra and their dependence on time and $R_{m}$. In Sec. VIII, we show how the formation, growth, and movement of plasmoids affect the energy spectrum. Finally, we present our conclusions.

II. NUMERICAL METHOD

Our calculation follows techniques that were described previously.\textsuperscript{44} The resistive MHD equations for our two-dimensional simulations are

\begin{align}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, \\
\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] &= -\nabla p + \mathbf{J} \times \mathbf{B}, \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{R_{m}} \nabla^{2} \mathbf{B}, \\
\frac{\rho^{2}}{\gamma - 1} \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) &= \frac{1}{R_{m}} f_{2}^{2}, \\
p &= \rho T, \\
\nabla \cdot \mathbf{B} &= 0.
\end{align}

Here $\rho$, $\mathbf{v}$, $\mathbf{B}$, $p$, $\mathbf{J}$, and $T$ are mass density, flow velocity, magnetic field, gas pressure, electric current density, and temperature in dimensionless units. These are defined by $\mathbf{B} = \mathbf{B}'/B_{0}$, $\rho = \rho'/\rho_{0}$, $p = p'/\left(B_{0}^{2}/\mu_{0}\right)$, $\mathbf{v} = \mathbf{v}/v_{0}$, $T = \beta_{0} T'/2T_{0}$, $J = J'/J_{0}$, $v_{0} = B_{0}/\sqrt{\mu_{0} \rho_{0}}$, $J_{0} = B_{0}/\mu_{0} L$, and $\beta_{0} = 2 \mu_{0} p_{0}/B_{0}^{2}$. Here, $\gamma = 5/3$, $\mu_{0}$, and $R_{m} = v_{0} L/\eta_{m}$ are the ratio of specific heats, the magnetic permeability of free space, and the magnetic Reynolds number based on the characteristic length $L$ and the velocity $v_{0}$, respectively. The magnetic diffusivity is $\eta_{m}$. The above characteristic variables are taken to be: $B_{0} = 10 G$, $T_{0} = 10^{8} K$, $\rho_{0} = 2.9 \times 10^{-15} g cm^{-3}$, and $L = 10^{5} km$. This leads to $\beta_{0} \approx 0.1$, $v_{0} \approx 525 km s^{-1}$, and $t_{0} \approx 190 s$.

We solve the above equations using the SHASTA code.\textsuperscript{45,46} The induction equation is written in terms of the magnetic vector potential, $\mathbf{A}$

\begin{align}
\frac{\partial \mathbf{A}}{\partial t} + \nabla \cdot (\mathbf{v} \times \mathbf{A}) &= \mathbf{A} \nabla \cdot \mathbf{v} + \mathbf{R}_{m}^{-1} \nabla^{2} \mathbf{A},
\end{align}

where $\mathbf{B} = \nabla \times \mathbf{A}$. In the two-dimensional case, this becomes $\mathbf{B} = \nabla \times A_{y}$ with the $y$-component of $\mathbf{A}$ and $y$ the unit vector in the $y$-direction. Specifically, we have replaced the two separate diffusive equations for $B_{x}$ and $B_{z}$ in Weber’s SHASTA code\textsuperscript{46} by a single diffusive equation for $A$, and there are two subroutines which compute $A$ from $\mathbf{B}$ and $\mathbf{B}$ from $A$, respectively.\textsuperscript{22}

The numerical experiment starts with a Kopp-Pneuman configuration which is in mechanical equilibrium, and is line-tied to the bottom boundary. The initial equilibrium is given by

\begin{align}
B_{x}(x, z, t = 0) &= x/|x|, \quad \text{for} \ |x| > w, \\
B_{z}(x, z, t = 0) &= \sin(\pi x/2w), \quad \text{for} \ |x| \leq w, \\
v(x, z) &= 0, \\
\rho(x, z) &= \frac{\beta_{0} + 1 - B_{0}^{2}}{\beta_{0}}, \\
p(x, z) &= T \rho(x, z, t = 0) = \beta_{0} \rho(x, z, t = 0)/2,
\end{align}

where $w$ is the half-width of the current sheet, and all parameters in Eq. (8) are dimensionless. The boundary conditions are as follows: the right and top sides of the simulation domain have open or free boundaries along which the plasma
and the magnetic flux are allowed to enter or exit freely, the left side is a symmetric boundary along the center of the current sheet, and the bottom boundary represents the solar surface at which the magnetic field is line-tied.

With uniform calculation mesh grids \((501 \times 1001)\), we perform a set of simulations with different magnetic Reynolds number \((R_m)\) ranging from 800 to \(3 \times 10^5\), and the same initial and boundary conditions. A small inflow perturbation equivalent to \(\delta v = 0.03 v_0\) is added to the initial configuration. The magnetic fields at two sides of the current sheet then move toward one another due to the attraction between fields of opposite polarity. This squeezes the sheet until it becomes thin enough that the tearing mode and other instabilities may take place. For high \(R_m\) cases, alternating magnetic islands and X-points appear in the sheet and move away from the location where they are formed.

### III. CRITICAL MAGNETIC REYNOLDS NUMBER

It is well known that the magnetic Reynolds number, \(R_m\), of the system is important for determining whether or not magnetic islands form. As reported in our previous studies, for low \(R_m\), reconnection quickly sets in through almost the whole current sheet with only a single X-point. In a high \(R_m\) environment, on the other hand, reconnection becomes difficult in a thick sheet and it takes time for the sheet to become thin enough via expelling the extra mass inside. Fast reconnection commences when magnetic islands or plasmoids form in the sheet. This implies the existence of a critical value of \(R_m\) below which no magnetic islands form in the sheet during reconnection. We denote this critical value of \(R_m\) as \(R_{mc}\). To look for \(R_{mc}\), we perform a set of numerical calculations in which \(R_m\) ranges from 800 to \(3 \times 10^5\). In all cases, the initial value of the plasma \(\beta\) outside the sheet is 0.1, and the evolution of the system starts with the same perturbation as described earlier.

Figure 1 shows the matter density distribution and the magnetic configuration as reconnection progresses for different \(R_m\). The left panel is for low \(R_m\) \((= 800)\), the middle one is for medium \(R_m\) \((= 1000)\), and the right one is for high \(R_m\) \((= 5 \times 10^4)\). During the whole evolutionary process of the sheet, only one plasmoid or magnetic island appears in the sheet for \(R_m = 1000\), and no plasmoids are observed for \(R_m < 1000\). This suggests that the value of \(R_{mc}\) in the case studied here is around 1000.

The value of \(R_{mc}\) obtained here is lower than those found by previous numerical works, for instance, \(10^4\) (see Ji and Daughton). This could be caused by the line-tied boundary or the amplitude of the initial perturbation. In our cases, the bottom boundary is line-tied so the plasma is prevented from moving downward freely. The downward motion instead stops at the top of the closed magnetic field lines, where the flare loop is believed to be. During this process, the interaction of plasma flow with the closed magnetic structure makes instabilities in the sheet occur more easily. The initial perturbation may also determine the occurrence of magnetic island formation. This is still an open question, and is worth further investigation.

### IV. PLASMOID FLUX, WIDTH, AND MASS DISTRIBUTION FUNCTIONS

The statistical properties of the plasmoid chain can be described using the plasmoid flux distribution function \(f(\psi)\). Here, \(\psi\) is the magnetic flux measured as the difference of magnetic vector potential \((A_y)\) between the O-point at the center of a given island and the nearest X-point. Following the practice of Uzdensky et al., we measure the cumulative plasmoid flux distribution function \(N(\psi)\), which

![FIG. 1. Density \(\rho\) (color scales) and magnetic flux (solid lines) contours as reconnection progresses in different \(R_m\) environments: (a) \(R_m = 800\), (b) \(R_m = 10^4\), and (c) \(R_m = 5 \times 10^4\).](image)
is the total number of plasmoids with a flux larger than $\psi$ along the current sheet. The distribution function is computed via $f(\psi) = -dN(\psi)/d\psi$, which is the plasmoid number per unit flux for a flux of $\psi$. In a two-ribbon flare current sheet, the plasmoid flux distribution function shows features of magnetic flux ejected during the reconnection process, in which the reconnected flux is ejected toward higher altitudes through the movement of plasmoids. Therefore, the flux distribution function displays the amount of the reconnected flux and scales of plasmoids during the CME/flare eruption.

The plasmoid flux distribution function $f(\psi)$ is displayed in Fig. 2. Plasmoids appearing in different time slices with an interval of $0.1 \, t_0$ are accumulated during the evolution of the current sheet, and three curves are for values of the magnetic Reynolds number ($R_m$) ranging from $1.58 \times 10^4$ to $2 \times 10^5$. Since the number of plasmoids appearing in low $R_m$ cases is insufficient for a statistical analysis, we consider results for high $R_m$ only. As shown in Fig. 2, $f(\psi)$ decreases overall with $\psi$, and is consistent with a power law distribution of $f(\psi) \sim \psi^{-2}$ for large $\psi$. For the cases of $R_m = 5 \times 10^4$ and $R_m = 1.58 \times 10^4$, these curves deviate from a power law and become shallower at small $\psi$. Since plasmoids with smaller scales occur infrequently during low $R_m$ simulations (see Fig. 3, we discuss this feature on the following paragraph), the number of small magnetic islands approaches an upper-limit that results in the drop of $f(\psi)$.

We also plot the size distribution function $f(w_x)$ in Fig. 3. Here, $w_x$ is the width of the plasmoid in the $x$-direction across the current sheet. The size of each plasmoid is calculated using the magnetic vector potential ($A_x$) profile in the $x$-direction. We introduce an artificial critical value $A_{xc}$ to identify the border of each plasmoid, along which $A_x$ decreases from the maximum ($A_{x0}$) in the center to this critical value ($A_{xc} = (A_{x0} - A_{xs})/e + A_{xs}$). Here, $A_{xs}$ is the magnetic vector potential of the nearest neighboring X-point and $e$ is Euler’s number. We then compute the size distribution function using the techniques performed to calculate $f(\psi)$. For the case of $R_m = 2 \times 10^5$, $f(w_x)$ shows a power law tendency that is similar to $f(\psi)$ except that it is steeper. However, for low $R_m$ cases, the value of $f(w_x)$ drops when $w_x$ decreases to roughly $0.04$ for $R_m = 5 \times 10^4$ and $0.02$ for $R_m = 1.58 \times 10^4$. This indicates that plasmoids grow quickly at low $R_m$ and exist for only a short time at small scales.

Similarly, we compute the mass distribution function of plasmoids. We identify the size of plasmoids in both directions along and across the current sheet and compute the amount of mass inside a plasmoid. The dashed straight line corresponds to $f(w_x) \sim w_x^{-3}$. We notice that $f(w_x)$ depends on the width of the plasmoid
The plasmoids are nearly circular around moids, which tend to be circular due to magnetic tension. Plasmoids correspond to older and more developed plas-islands, and they show a power law distribution of high detail the physical picture that charged particles could be sheet and have low mass and low magnetic flux. In addition, a group of plasmoids corresponds to the initial phase of develop-ment. Therefore, it is reasonable to expect the curve of \( f(m) \) to be different from \( f(w) \). In addition, we discuss the relationship between the length to width ratio of plasmoids and the mass distribution in the following paragraph.

Figure 5 shows the distribution of plasmoid width along two dimensions for the case of \( R_m = 5 \times 10^4 \). Here, \( w_z \) is the width of the plasmoid in the \( z \)-direction along the current sheet and \( w_x \) is for the \( x \)-direction. The color indicates the mass of plasmoids. This plot shows the nature of the shape of plasmoids and mass distributions: dominant and heavy plasmoids are in a triangular region. First, most heavy plasmoids are located around, but above the diagonal line representing a length to width ratio of \( w_z/w_x = 1 \). These plasmoids correspond to older and more developed plasmoids, which tend to be circular due to magnetic tension. The plasmoids are nearly circular around \( w_z = w_x = 0.1 \), but there is a large range of elongations from \( w_z/w_x = 0.06 \) to \( w_z/w_x = 0.7 \) for low mass plasmoids with \( w_x \) near 0.01. This group of plasmoids corresponds to the initial phase of development when plasmoids are elongated along the current sheet and have low mass and low magnetic flux. In addition, Fig. 5 shows that some plasmoids have a much larger width than others with the same mass and are located below the diagonal line. This region corresponds to plasmoids that are impacting the top of the flare loops. These magnetic islands are pressed flat so that the width of each plasmoid can become longer than the length.

The circularization of plasmoids is important for particle acceleration in some models.\(^{38-51}\) Drake et al.\(^{50}\) discussed in detail the physical picture that charged particles could be efficiently accelerated through reflection in contracting islands, and they show a power law distribution of high energy particles during the contraction of islands in PIC simulations. In addition, Kowal et al.\(^{31}\) performed MHD simulations and investigated the acceleration of test particles during island contraction. They find that first-order Fermi processes occur in the contracting islands during merging. According to these studies, it is clear that the circularization of islands is an integral part of particle acceleration. Furthermore, our calculations show that the circularization of islands is frequent during the development of plasmoids. Figure 5 shows that plasmoids mostly tend to get circular during the growth process. However, most plasmoids are located above the diagonal line and do not become purely circular. This indicates that plasmoids continually undergo circularization. Before plasmoids reach the diagonal line in Fig. 5, they can continually accelerate particles according to the model of Drake et al.\(^{50}\) Therefore, our results are consistent with abundant high energy particles being accelerated efficiently in fragmented flare current sheets.

According to the above distributions of flux, size, and mass, we can compare our calculations with the predictions of models.\(^{30,35}\) The width distribution of plasmoids is roughly consistent with the prediction by Uzdensky et al.\(^{30}\) in that it follows \( f(w_z) \sim w_z^{-2} \). For large scale plasmoids, the shape of the flux distribution function \( f(\psi) \) is consistent with following \( \sim \psi^{-2} \) as well, while the mass distribution function \( f(m) \) is steeper than an inverse square power law. On the other hand, for small scale plasmoids, both \( f(\psi) \) and \( f(m) \) are flatter than an inverse square power law and also deviate from the prediction reported by Fermo et al.\(^{15}\) of \( \sim \psi^{-1} \) and calculations reported by Huang and Bhattacharjee.\(^{38}\) The difference between our results and predictions of models could be due to the numerical setup, in which our calculations are limited to larger scales than previous models\(^{30,35}\) and simulations.\(^{37,38}\) In our cases, the magnetic Reynolds number \( (R_m) \) is less than previous simulations reported by Huang and Bhattacharjee,\(^{38}\) which results in the absence of smaller scale plasmoids in our present calculations. At small scales, more complex evolution of flux could occur, including, for example, more tiny plasmoids during the initial period of formation. We shall carefully discuss this feature via considering abundant tiny plasmoids in the future. The line-tied boundary condition on the bottom boundary in our calculations also affects movements of plasmoids, which is different from models with a symmetric configuration.\(^{30,35}\) In addition, the models deal with growth and merging processes of magnetic islands,\(^{30,35}\) while plasmoids exhibit complex changes in present simulations. For instance, elongated plasmoids become more circular after they form, which may affect significantly the flux and width distribution functions.

V. SPECTRAL ANALYSIS OF THE CURRENT SHEET

To look into the energy distribution of magnetic islands occurring in the reconnecting current sheet, we perform a spectral analysis in one dimension along the current sheet (\( z \)-direction) for the magnetic energy and the kinetic energy using the Fourier transform method. In the following paragraph, we analyze the case of \( R_m = 5 \times 10^4 \) in which multiple magnetic islands and plasmoids appear and develop.
Figure 6(a) shows the distribution of the average magnetic energy density ($E_m$) in a strip along the $z$-axis five cells wide at two given times $t = 26.5$ and $t = 36.6$. At $t = 36.6$, the total electric current density is close to its maximum. As shown in Fig. 6(a), the black line shows that there are peaks in the magnetic energy profile along the $z$-direction, which indicate magnetic islands or plasmoids. We apply the Fourier transform to this distribution of $E_m$, and plot the distribution of magnetic energy in wave number $k$ space in Fig. 6(b), which is known as the Fourier power spectrum profile. Here, the wave number $k$ is normalized to the reciprocal of the characteristic length of the system ($L = 10^3$ km). The magnetic energy spectrum for small $k$ is dominated by large structures, while for large $k$ the energy spectrum is dominated by small structures such as magnetic islands.

For intermediate wave numbers ($10 < k < 100$), which covers most of the scales of magnetic islands in our simulations, the spectrum could be fit using a power law. Because the empirical data from the Fourier transform include non-ignorable noise, linear regression on doubly logarithmic axes yields systematic errors when calculating the power law index. We therefore use another fitting strategy in the following calculations. First, we fit the spectrum using the function $E = a_0 k^{-7}$ while assuming the error bars on each data point are identical. Then using the residuals of the previous fit as the error bars, we perform a subsequent fit and perform the computation recursively. This approach assumes that the error is proportional to the amplitude of the power law, which is reasonable in our case. In the absence of well-defined error bars, robust statistical analyses are not performed in this paper. However, according to results of the fitting, we interpret the power spectra as power laws. The solid straight line in Fig. 6(b) gives the fit to the simulation data, and the index for the magnetic energy spectrum ($\gamma_m$) is 2.92 at time 36.6. We also notice that the fitting depends sensitively on the choice of lower bound on wave number according to the discussion by Clauset et al., so we compare the fitting result using higher cut off wave number ($15 < k < 100$). The average difference of power indices between two cases is roughly less than 0.3, which implies that using 10 as the lower bound on the $k$ axis is reasonable in the following analyses.

The red dashed line in Fig. 6(a) shows the situation at $t = 26.5$ before magnetic islands developed inside the sheet, in which the magnetic energy decreases smoothly in the outflow direction. Although the magnetic islands have not developed by the earlier time, we also could calculate the power spectrum using similar Fourier analysis. In Fig. 6(b), the red dashed line shows the magnetic energy distributions in wave number space, and the fitted power law index is $\gamma_m = 1.98$. Comparing these two power spectrum profiles, the magnetic energy increases significantly in the middle and large wave number ranges at the later time ($t = 36.6$). This implies that more small and intermediate scale magnetic structures appear in this time during the development of magnetic islands.

The kinetic energy of the plasma, including that of the flow and the plasmoids, shows similar behavior. For the same value of $R_m$ as that used for Fig. 6, Fig. 7 plots the plasma kinetic energy along the $z$-axis (Fig. 7(a)) and the corresponding Fourier power spectrum (Fig. 7(b)). A power law function is used to fit the simulation data with an index $\gamma_k$ of 2.63 at $t = 36.6$ and 1.95 at $t = 26.5$. Here, $\gamma_k$ is the index for the kinetic energy spectrum.

**VI. VARIATION OF THE ENERGY SPECTRUM VERSUS TIME**

In the process of reconnection, properties of the energy spectrum also change with time. As we noticed earlier, the evolution of the current sheet consists of two stages with the appearance of the plasmoids as the turning point. In the early phase, magnetic reconnection occurs slowly and the reconnection outflow moves smoothly along the sheet. The evolution enters the second phase when various instabilities begin to develop so that magnetic islands begin to dominate the reconnection process. In this stage, the reconnection rate defined as the ratio of the inflow speed to the local Alfvén speed $M_A = V_{in}/V_A$ rapidly increases from a small value ($<0.01$) to 0.1 after plasmoids appear. Therefore, it is worth investigating the properties of the energy spectrum at different times as well.
Duplicating our techniques performed in Sec. V for the spectral analyses, we are able to measure the spectral indices at different times for the spectra of magnetic and kinetic energies. Figure 8(a) plots variations of indices ($\gamma_m$ and $\gamma_k$) for $R_m = 5 \times 10^4$. Both curves indicate that the spectral index remains roughly constant such that $\gamma_m = 1.98$ and $\gamma_k = 1.95$ in the earlier stage ($t < 29.5$). In the second stage ($t > 29.5$, see the arrows in the panel), on the other hand, both indices show a tremendous increase associated with the formation of a large group of magnetic islands. Following the increase, the indices show apparent oscillating features, which can be ascribed to the successive production of magnetic islands. The corresponding curves fluctuate around 3.02 for $\gamma_m$ and 2.79 for $\gamma_k$, respectively.

The increase of $\gamma_m$ and $\gamma_k$ in the second stage occurs when the instability emerges. This can be examined by looking into how the total magnetic energy $E_m$ and the total kinetic energy $E_k$ change with time. Figure 8(b) plots variations of the two energies versus time. Both $E_m$ and $E_k$ undergo gradual evolution after the experiment commences such that $E_m$ decreases slowly and $E_k$ increases slowly before $t = 29.5$. After the instability is initiated, say after $t = 29.5$ as specified by the arrow, $E_m$ quickly drops from $2.0 \times 10^4$ to $1.5 \times 10^4$, and $E_k$ rapidly increases from $1.6 \times 10^3$ to $2.1 \times 10^3$ within the same time interval. A decrease in $E_k$ then follows as a result of the instability.

We need to note here that our experiment indicates that the first O-type neutral point appears at $t = 26.8$, which is the signature of the development of the instability in the current sheet (see Shen et al.44). However, multiple plasmoids form after time 29.5. As shown by comparing the curves in Figs. 8(a) and 8(b), the formation of the first O-point (or plasmoid) does not directly relate to the dramatic change in the spectral index but to the appearance of multiple plasmoids. This is because it takes time for the non-linear reconnection process to fully develop. A similar phenomenon has also been noticed by Shibata and Tanuma53 in theoretical studies and by Mei et al.54 in numerical experiments. The fluctuating behavior of $\gamma_m$ (and $\gamma_k$) is an apparent characteristic of magnetic reconnection with magnetic islands. Similar oscillating features observed in the magnetic reconnection rate were also observed in recent numerical simulations.28,33,44,54,55

FIG. 7. Same as Fig. 6, but for the kinetic energy.

FIG. 8. (a) Variations of $\gamma_m$ and $\gamma_k$ versus time for $R_m = 5 \times 10^4$. The solid line is for $\gamma_m$ and the dashed line is for $\gamma_k$. (b) Variations of $E_m$ and $E_k$ of simulation box versus time. The solid line is for $E_m$ and the dashed line is for $E_k$. The arrows in both panels specify the time when the non-linear stage begins.
VII. VARIATIONS OF THE ENERGY SPECTRUM VERSUS $R_m$

To better understand the impact of interactions among the multiple plasmoids on the spectral indices, we study the dependence of $\gamma_m$ and $\gamma_k$ on $R_m$ in this section. As indicated by previous simulations and related theoretical studies, many plasmoids always appear in the current sheet in a high $R_m$ environment, which yields frequent interactions among plasmoids.

For $R_m$ ranging from $4 \times 10^3$ to $3 \times 10^5$, we study the experimental data obtained in the period when multiple plasmoids appear and interactions among them occur. For given $R_m$, we estimate $\gamma_m$ and $\gamma_k$ in a certain period, say $\Delta t \approx 2 \sim 3 \Delta_t$, in which the current density in the sheet reaches or approaches its maximum. At this time, the sheet becomes very thin and various instabilities including the tearing mode have thoroughly developed. We calculate the average spectral indices, denoted as $\bar{\gamma}_m$ and $\bar{\gamma}_k$, respectively, during the whole evolutionary process to look into the instant cascading of energy in a certain period.

Figure 9(a) shows variations of $\bar{\gamma}_m$ and the maximum of $\gamma_m$, $\gamma_{m,max}$, versus $R_m$. An overall feature of these two curves is that both values of the index decrease roughly with $R_m$, but the change slows down in the range of high $R_m$, say $R_m > 2 \times 10^4$. Furthermore, we notice that $\gamma_{m,max}$ is larger than 3.5 and can exceed 5.0, while $\bar{\gamma}_m$ changes between 3.88 and 3.16. Another feature is that the amplitude of the alternation in $\gamma_{m,max}$ is smaller than that in $\bar{\gamma}_m$.

We plot $\gamma_k$ and $\gamma_{k,max}$, which denote the average and the maximum values of $\gamma_k$, respectively, against $R_m$ in Fig. 9(b). Different behaviors of $\gamma_k$ and $\gamma_{k,max}$ from those of $\gamma_m$ and $\gamma_{m,max}$ are seen clearly. $\gamma_{k,max}$ and $\gamma_k$ both displays oscillating features of large amplitude. On the other hand, the change of $\gamma_k$ with $R_m$ is different than that of $\gamma_m$. The profile of $\gamma_k$ shows a tendency to increase with $R_m$. Therefore, the average power index ($\bar{\gamma}_k$) tends to exceed 3.0 for large $R_m$.

The fact that both $\gamma_m$ and $\gamma_{m,max}$ manifest oscillating features indicates that the process is nonlinear and multiscale. These features are caused by the interaction among multiscale magnetic islands, such as colliding and merging, and by the interaction between magnetic islands and the ambient plasma.

VIII. PHYSICAL MECHANISMS RESPONSIBLE FOR CHANGES IN MAGNETIC ENERGY SPECTRA

In this section, we discuss the physical processes that are most likely to be responsible for changes in the magnetic energy spectrum or power law index. We focus on the dynamics of islands because they contain the majority of the magnetic energy. When a process acting on all scales takes energy from short length scales and moves it to longer length scales, it makes the power spectrum steeper. Conversely, processes that take energy away from long length scales and put it into shorter length scales make the power spectrum shallower. The shape of the power law spectrum depends on the combination of all of these processes.

The growth of islands moves energy from small scales to large scales, and therefore makes the power spectrum locally steeper. Islands grow through two processes. The first process is normal growth through the accumulation of flux that is reconnected at surrounding X-points, and the second process is island merging. When a new magnetic island forms, it moves along the sheet and develops rapidly to scales larger than the width of the current sheet. That causes the magnetic energy to move from a small scale to a larger scale. Figure 10 shows an example of these growth processes. From $t = 32.5$ to $t = 33.0$, a new magnetic island formed at $z = 0.7$ along the sheet, and it develops to a sizable plasmoid until $t = 33.0$. The distribution of magnetic energy is plotted on Fig. 10(b), in which this new island could be seen as a clear peak on the energy profile at the final time $t = 33.0$. This means the magnetic energy associated with the scale of this new island increases significantly until $t = 33.0$. 

![FIG. 9. Dependence of the Fourier power spectrum indices on $R_m$: (a) $\gamma_m$ versus $R_m$, and (b) $\gamma_k$ versus $R_m$. The solid lines are for the spectral index averaged over a given time interval when the total electric current density is close to its maximum, and the dashed lines are for the index at maximum.](image-url)
FIG. 10. Magnetic energy spectrum becomes steeper with the growth of magnetic islands. (a) The distributions of density $\rho$ along the center of the current at different times, which shows that the new magnetic islands form and grow rapidly during this period; (b) The distribution of magnetic energy at $t = 32.5$ and $t = 33.0$; (c) and (d) are for magnetic energy spectrum at initial and final time.

FIG. 11. Magnetic energy spectrum becomes steeper with the merging process of magnetic islands. (a)–(d) are same as Fig. 10.
As shown in Figs. 10(c) and 10(d), the magnetic energy for intermediate wave numbers increases and makes the power spectrum steeper from $c_m = 3.21$ to $c_m = 4.33$.

Figure 11 shows how the magnetic energy spectrum changes during the merging process of two plasmoids. From $t = 40.2$ to $t = 41.0$, two upward moving islands merge and finally form a new larger island located at $z = 1.7$. The magnetic energy moves to the larger scale after the merging process, and the spectral index increases from $\gamma_m = 2.58$ to $\gamma_m = 3.33$. However, the merging process itself is complicated because changing the separation between islands also changes the power spectrum. When islands move closer together, small structures could be created because of the compression, which makes the power spectrum shallower. After the merging process, the power spectrum becomes steeper than before.

Islands leaving the system will also affect the power spectrum by reducing the power on the scales of the islands that leave. For instance, if large islands leave, then the power spectrum will become shallower. This is expected to be the case if the islands that leave are preferentially older and therefore larger islands. Because of the open boundary conditions in our calculations, it frequently occurs that larger islands exit the system through the top boundary. Figure 12 shows an example where an island leaves the system at $t = 42.1$. The spectral index decreases significantly to $\gamma_m = 2.59$ at $t = 42.4$. During this period, no clear island forms in the other parts of the current sheet, which implies that the decreasing of index $\gamma_m$ is the result of the older larger island exiting the system.

The power spectrum will also be affected by changes in the structure of current sheets, oscillations in the size of islands, and changes in the structures of islands. When a new island (or equivalently, a new X-point/O-point pair) forms, the pre-existing current sheet is broken up into shorter segments. This takes energy from large scales and moves it to small scales, which makes the spectrum shallower. New islands are generally elongated along the outflow direction, and magnetic tension acts to circularize them. When islands contract, this makes the power spectrum shallower. However, fragmentation of the current sheet normally occurs with growth of magnetic islands which quickly develop and move along the sheet. Therefore, the magnetic energy spectrum is significantly affected by the subsequent dynamical development of magnetic islands.

IX. CONCLUSION

In this work, we perform a set of numerical experiments for magnetic reconnection taking place in the two-ribbon flare process. Applying line-tied and open boundary conditions to the bottom and the other boundaries, respectively, and an artificial perturbation to the open boundary at the beginning, we investigated the evolution of the current sheet for different $R_m$, and studied properties of the instability and the consequent non-linear dynamics occurring in the current sheet. A critical value of $R_m$ exists, below which magnetic reconnection progresses smoothly and gradually, and no instability and turbulence take place. In the case we studied
here, the critical value of $R_m$ is $R_m = 10^3$. This value is smaller than those of previous numerical experiments, as well as that expected from the theory of plasma instabilities. This could be due to either the line-tied boundary, or strong initial perturbation, or both.

When $R_m$ exceeds $R_{mc}$, instabilities and magnetic islands were observed to occur in the sheet. Applying the Fourier transformation to distributions of both the magnetic energy and the kinetic energy in the sheet as reconnection progresses, we obtained the Fourier power spectrum for both the energy distributions. We interpret the distributions of either the magnetic energy ($\gamma_m^c$) or the kinetic energy ($\gamma_k^c$) as power laws. In addition, the probability distribution function of magnetic field increments along the current sheet shows a non-Gaussian distribution, which will be considered in future work. We also note that our numerical experiments were performed in a 2-D configuration, and the result could be different in 3-D cases. This is an interesting and open question, and is worth studying further.

The flux and size distribution function of plasmoids roughly follow the inverse square power law at large scales, which is consistent with predictions of Uzdensky et al. On the other hand, the mass distribution function is steeper than the inverse square power law at large scales and shallower at small scales. The distribution of the length to width ratio shows that plasmoids are highly asymmetric in the initial phase of formation. They tend to become more circular as plasmoids grow.

Both $\gamma_m$ and $\gamma_k$ are functions of time. In the early stage of reconnection, both $\gamma_m$ and $\gamma_k$ are small (<2.0) and change very slowly until magnetic islands appear in the sheet, which marks the occurrence of the instability and the beginning of the second stage. Then a tremendous increase in the two indices was observed, followed by an oscillation with an average of about 3, indicating the start of the second stage.

The indices $\gamma_m$ and $\gamma_k$ also depend on $R_m$. In the case of small $R_m$, they display oscillations, and for large $R_m$, on the other hand, $\gamma_m$ decreases to approach a constant and $\gamma_k$ continues to increase but also seems to approach to constant. The physics behind such dependence is not clear yet, and may require further study.

The dynamics of magnetic islands cause changes of magnetic energy spectra with time. The physical mechanisms include the formation, growth, and movement of islands. The growth of islands moves energy from small scales to large scales to make the power spectrum steeper. By contrast, the power spectrum becomes shallower when developed islands leave the system.

ACKNOWLEDGMENTS

We think the anonymous referee for very useful comments. This research is supported by NASA Grant NNX11AB61G, NASA Contract NNN0F080FC, and NSF SHINE Grant AGS-1156076 to the Smithsonian Astrophysical Observatory. This work was also supported by Program 973 Grants 2011CB811403 and 2013CBA01503, NSFC Grant 11273055, and CAS Grant KJCX2-EW-T07.

56 J. Büchner, private communication (2012).