The plasmoid instability during asymmetric inflow magnetic reconnection

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Magnetic reconnection is the breaking and rejoining of magnetic field lines in a highly conducting plasma.

The classical Sweet-Parker model predicts that the reconnection rate scales as $S^{-1/2}$ (where $S \sim \frac{LV_A}{\eta}$). Too slow to explain solar flares and fast reconnection elsewhere.

In recent years, it has been discovered that high aspect ratio current sheets are susceptible to the formation of plasmoids (Loureiro et al. 2007; Huang et al. 2011).

- Breaks up the current sheet into a chain of X-lines and islands.
- The reconnection rate asymptotes at $\sim 0.01$ for large $S$.

The role of this instability may be to bring structure down to small enough scales that collisionless effects become important (Shepherd & Cassak 2010).
Most simulations of the plasmoid instability assume reconnection with symmetric upstream fields
  ▶ Simplifies computing and analysis
  ▶ Plasmoids and outflows interact in one dimension

Asymmetry affects the scaling and dynamics of the plasmoid instability

In 3D, flux ropes twist and writhe and sometimes bounce off each other instead of merging
  ▶ Asymmetric inflow reconnection simulations offer clues to 3D dynamics
Asymmetric Magnetic Reconnection

- **Asymmetric inflow reconnection** occurs when the upstream magnetic fields and/or plasma parameters differ
  - Dayside magnetopause
  - Tearing in tokamaks, RFPs, and other confined plasmas
  - Merging of unequal flux ropes
  - ‘Pull’ reconnection in MRX

- **Asymmetric outflow reconnection** occurs, for example, when outflow in one direction is impeded
  - Flare/CME current sheets
  - Planetary magnetotails
  - Spheromak merging
  - ‘Push’ reconnection in MRX

- Asymmetric inflow reconnection often occurs at the boundaries between different plasmas

- Asymmetric outflow reconnection often occurs during explosive events
NIMROD solves the equations of extended MHD using a finite element formulation (Sovinec et al. 2004, 2010)

In dimensionless form, the resistive MHD equations used for these simulations are

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times (\eta \mathbf{J} - \mathbf{V} \times \mathbf{B}) + \kappa_{\text{divb}} \nabla \nabla \cdot \mathbf{B} \quad (1)
\]

\[
\mathbf{J} = \nabla \times \mathbf{B} \quad (2)
\]

\[
\nabla \cdot \mathbf{B} = 0 \quad (3)
\]

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \rho \nu \nabla \mathbf{V} \quad (4)
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = \nabla \cdot D \nabla \rho \quad (5)
\]

\[
\frac{\rho}{\gamma - 1} \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -\frac{\rho}{2} \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{q} + Q \quad (6)
\]

Divergence cleaning is used to prevent the accumulation of divergence error
Reconnecting magnetic fields are asymmetric:

\[ B_y(x) = \frac{B_0}{1 + b} \tanh \left( \frac{x}{\delta_0} - b \right) \]  

A small number of localized initial magnetic perturbations placed asymmetrically along \( z = 0 \) near center of domain

Symmetric case:
- \( \{B_1, B_2\} = \{1.00, 1.00\} \); \( S_{Ah} \sim 1 \times 10^5 \); \( V_{Ah} = 1.0 \)

Asymmetric case:
- \( \{B_1, B_2\} = \{1.00, 0.25\} \); \( S_{Ah} \sim 5 \times 10^4 \); \( V_{Ah} = 0.5 \)

Uniform initial density

\( \beta_0 = 1 \) in higher magnetic field upstream region

Domain: \(-150 \leq x \leq 150, -16 \leq z \leq 16 \)

Boundary conditions: periodic along outflow direction and conducting wall along inflow direction
Mesh packing needed over longer portion of inflow direction
  ▶ X-lines drift toward strong magnetic field upstream region
  ▶ Somewhat less resolution required along outflow direction than in symmetric case
  ▶ Higher resolution required in weak $\mathbf{B}$ upstream region than in strong $\mathbf{B}$ upstream region

Preliminary simulations showed sloshing/oscillatory behavior
  ▶ Symmetric perturbations led to asymmetric magnetic pressure imbalance
  ▶ Resolved by using weak, localized perturbations and increasing the size of the domain along the inflow direction
Plasmoid instability: symmetric inflow
Plasmoid instability: asymmetric inflow

- Magnetic Flux
- Current density, $J_y$ (range: $-1.61$ to $1.85$)
- Outflow velocity, $V_x$ (range: $±0.32$)
- Inflow velocity, $V_z$ (range: $±0.14$)
- Vorticity, $(\nabla \times V)_y$ (range: $±0.47$)
Key features of symmetric inflow simulation

- X-points and O-points all located along $z = 0$
  - Makes it easy to find nulls
- X-lines often located near one exit of each current sheet
  - Characteristic single-wedge shape
- There is net plasma flow across X-lines
  - Flow stagnation points not co-located with X-line
  - The velocity of each X-line differs from the plasma flow velocity at each X-line (see Murphy 2010)
- Outflow jets impact islands directly
  - No net vorticity in islands and downstream regions
  - Less noticeable turbulence in downstream regions
- Outflow velocity $\sim 5/6$ of Alfvén speed
Key features of asymmetric inflow simulation

- Maximum outflow velocity is $\sim 2/3$ of $V_{Ah}$
- Current sheets thicker than symmetric case
- X-lines vary in position along inflow direction
- Islands develop preferentially into weak $B$ upstream region
- Outflow jets impact islands obliquely
  - Islands advected outward less efficiently
  - Net vorticity develops in each magnetic islands
- Downstream region is turbulent
  - Plasmoids impacting and merging with downstream island
  - Several X-points and O-points
- Very little happening in strong $B$ upstream region
  - Less resolution needed than in weak $B$ upstream region
- Secondary reconnection events (when islands merge) have asymmetric inflow and outflow
The asymmetric case shows little enhancement in the reconnection rate from the predicted value.

Use formulae from Cassak & Shay (2007); Birn et al. (2011):

\[ E_{\text{predict}} = \sqrt{\eta \frac{V_{Ah}}{L}} B_L B_R \]

\[ t_{Ah} = \frac{L}{V_{Ah}} \]

\[ L = 100 \]

Note: \( S_{Ah} \) is lower by a factor of two for the asymmetric case.
What insights do these simulations provide for the 3D plasmoid instability?

- Daughton et al. (2011): plasmoids in 3D will be complicated flux rope structures
- Outflow jets will generally impact flux ropes obliquely
  - Momentum transport from outflow jets to flux ropes may be less efficient
  - Merging between colliding flux ropes may be incomplete
- Important questions:
  - How does the plasmoid instability behave in 3D?
    - What is the reconnection rate? Is it 0.01 or 0.1?
  - How do reconnection sites interact in 3D?
  - What mistakes are we making by using 2D simulations to interpret fundamentally 3D behavior?
Murphy (2010) derived an exact expression for the rate of X-line retreat when it is restricted to 1D

\[
\frac{dx_n}{dt} = \left. \frac{\partial E_y}{\partial x} \middle/ \frac{\partial B_z}{\partial x} \right|_{x_n} = V_x (x_n) - \eta \left[ \frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial z^2} \right] x_n
\]

The 3D equivalent for the motion of isolated magnetic nulls is

\[
\frac{dx_n}{dt} = (\nabla B)^{-1} \nabla \times E = V (x_n) - \left[ \eta (\nabla B)^{-1} \nabla^2 B \right] x_n
\]

This provides insight into how nulls form, move, and disappear:

- Plasma flow across nulls allowed by resistive diffusion
- When the Jacobian matrix $\nabla B$ is singular, nulls are either appearing or disappearing
- Newly formed null-null pairs initially move apart very quickly
- Allows convenient tracking of nulls in 2D and 3D simulations
Conclusions

- We compare two simulations of the plasmoid instability with symmetric and asymmetric upstream magnetic fields.
- Features of the asymmetric simulation include:
  - X-line positions not all at same location along inflow direction
  - Islands develop into the weak $B$ upstream region
  - Outflow jets impact islands obliquely
    - Less efficient outward advection of islands
    - Circulation within each island
  - Turbulence in the downstream region
  - Broader current sheets than the symmetric case
  - The reconnection rate is not greatly enhanced above the predicted value for asymmetric reconnection without plasmoids
- We have derived an exact expression describing the motion of magnetic nulls in 3D.
Future Work

- Scaling study of asymmetric inflow plasmoid instability
  - How does asymmetry affect the onset criterion?
    - Is it a function of $S_{Ah} = \frac{LV_{Ah}}{\eta}$?
    - Is the reconnection rate significantly enhanced above the Cassak-Shay prediction as in the symmetric case?
- 3D simulations of $\geq 2$ competing reconnection sites
- Asymptotic matching analysis to determine the onset criterion and properties of the linear asymmetric plasmoid instability
  - Anybody interested?
- Investigate the role of additional terms in the generalized Ohm’s law on the 3D motion of nulls