ON THE SNOW LINE IN DUSTY PROTOPLANETARY DISKS

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ABSTRACT

C. Hayashi prescribed a "minimum-mass solar nebula," which contained just enough material to make the planets. This prescription, which has been widely used in constructing scenarios for planet formation, proposed that ice will condense when the temperature falls below 170 K (the "snow line"). In Hayashi's model, that occurred at 2.7 AU. It is usually assumed that the cores of the giant planets (e.g., Jupiter) form beyond the snow line. The snow line, in Hayashi's model, is where the temperature of a black body that absorbed direct sunlight and reradiated as much as it absorbed would be 170 K. Since Hayashi, there have been a series of more detailed models of the absorption by dust of the stellar radiation and of accretional heating, which alter the location of the snow line. We have attempted a "self-consistent" model of a T Tauri disk in the sense that we used dust properties and calculated surface temperatures that matched observed disks. We then calculated the midplane temperature for those disks, with no accretional heating or with small ($\leq 10^{-8}$ $M_\odot$ yr^{-1}) accretion rates. (Larger accretion rates can push the snow line out to beyond 4 AU but do not match the observed disks.) Our models bring the snow line in to the neighborhood of 1 AU—not far enough to explain the close planetary companions to other stars, but much closer than in recent starting lines for orbit migration scenarios.

Subject headings: accretion, accretion disks — dust, extinction — interplanetary medium — planetary systems — radiative transfer — stars: pre–main-sequence

1. INTRODUCTION

Planetary systems are formed from rotating protoplanetary disks, which are the evolved phase of circumstellar disks produced during the collapse of a protostellar cloud with some angular momentum.

A standard model of such a protoplanetary disk is that of a steady state disk in vertical hydrostatic equilibrium, with gas and dust fully mixed and thermally coupled (Kenyon & Hartmann 1987). Such a disk is flared, not flat, but still geometrically thin in the sense defined by Pringle (1981). The disk intercepts a significant amount of radiation from the central star, but other heating sources (e.g., viscous dissipation) can be more important. If dissipation due to mass accretion is high, it becomes the main source of heating. Such are the protoplanetary disks envisioned by Boss (1996, 1998), which have relatively hot (midplane temperature $T_m > 1200$ K) inner regions due to mass accretion rates of $\sim 10^{-6}$–10^{-5} $M_\odot$ yr^{-1}. However, typical T Tauri disks of age ~ 1 Myr seem to have much lower mass accretion rates ($\leq 10^{-8}$ $M_\odot$ yr^{-1}) with all other characteristics of protoplanetary disks (Hartmann et al. 1998; D'Alessio et al. 1998). For disks of such low accretion rates, stellar irradiation becomes increasingly the dominant source of heating, to the limit of a passive disk modeled by Chiang & Goldreich (1997). In our paper we will confine our attention to the latter case, without entering the discussion about mass accretion rates.

The optical depth in the midplane of the disk is very high in the radial direction, hence the temperature structure there is governed by the reprocessed irradiation of the disk surface. This is the case of a passive disk (no accretion). At some point along the radial direction, the temperature in the midplane would drop below the ice sublimation level—Hayashi (1981) called it the "snow line."

In this paper we revisit the calculation of the snow line for a protosolar protoplanetary disk, given its special role in the process of planet formation. We place particular empha-

sis on the issues involved in treating the radiative transfer and the dust properties.

2. THE MODEL

Our model is that of a star surrounded by a flared disk. In this paper we have chosen two examples: a passive disk and a disk with a 10^{-8} $M_\odot$ yr^{-1} accretion rate. Both have the same central star of effective temperature $T_* = 4000$ K, mass $M_* = 0.5$ $M_\odot$, and radius $R_* = 2.5$ $R_\odot$. Thus they correspond to the examples used by Chiang & Goldreich (1997) and D'Alessio et al. (1998), respectively. Our disk has a surface gas mass density $\Sigma = r^{-3/2}\Sigma_0$, with r in AU and $\Sigma_0 = 10^3$ g cm^{-2} for our standard minimum-mass solar nebula model; we varied Σ_0 between 10^2 and 10^4 g cm^{-2} to explore the effect of disk mass on the results.

The emergent spectrum of the star is calculated with a stellar model atmosphere code with Kurucz (1992) line lists and opacities. The disk intercepts the stellar radiation $F_{irr}(r)$ at a small grazing angle $\phi(r)$ (defined in § 5). The emergent stellar spectrum is input into a code which solves the continuum radiative transfer problem for a dusty envelope. The solution is a general spherical geometry solution with a modification of the equations to a section corresponding to a flared disk (see Menshchikov & Henning 1997 for a similar approach). In that sense, the radiative transfer is solved essentially in one dimension (vertically), as opposed to a full-scale consistent two-dimensional case. The appeal of our approach is in the detailed radiative transfer allowed by the one-dimensional scheme.

The continuum radiative transfer problem for a dusty envelope is solved with the method developed by Ivezić & Elitzur (1997). The scale invariance applied in the method is practically useful when the absorption coefficient is independent of intensity, which is the case of dust continuum radiation. The energy density is computed for all radial grid points through matrix inversion, i.e., a direct solution to the full scattering problem. This is both very fast and accurate

at high optical depths. Note that in our calculations (at $r \geq 0.1$ AU) the temperatures never exceed 1500–1800 K in the disk and we do not consider dust sublimation; the dust is present at all times and is the dominant opacity source. As in the detailed work by Calvet et al. (1991) and more recently by D'Alessio et al. (1998), the frequency ranges of scattering and emission can be treated separately.

For the disk with mass accretion, the energy rate per unit volume generated locally by viscous stress is given by $2.25\alpha P(z)\Omega(r)$, where the turbulent viscosity coefficient is $\nu = \alpha c_s^2 \Omega^{-1}$, Ω is the Keplerian angular velocity, $c_s^2 = P\rho^{-1}$ is the sound speed, and a standard value for $\alpha = 0.01$ is used. The net flux produced by viscous dissipation, $F_{\rm vis}$, is the only term to balance $F_{\rm rad}$. Unlike D'Alessio et al. (1998), we have ignored the flux produced by energetic particle ionization. Then we have the standard relation (see Bell et al. 1997), which holds true for the interior of the disk where accretion heating occurs:

$$\sigma T_{\rm vis}^4 = \frac{3\dot{M}GM_*}{8\pi r^3}\left[1 - \left(\frac{R_*}{r}\right)^{1/2}\right],$$

where $\dot{M}$ is the mass accretion rate, and M_* and R_* are the stellar mass and radius.

3. THE DUST

The properties of the dust affect the wavelength dependence of scattering and absorption efficiencies. The temperature in the midplane is sensitive to the dust scattering albedo (ratio of scattering to total opacity)—a higher albedo would reduce the absorbed stellar flux. As with our choice of mass accretion rates, we will use dust grains with properties that best describe the disks of T Tauri stars.

The modeling of circumstellar disks has always applied dust grain properties derived from the interstellar medium. Most commonly used have been the grain parameters of the Mathis, Rumpl, & Nordsieck (1977) distribution with optical constants from Draine & Lee (1984). However, recent work on spectral distributions (Whitney, Kenyon, & Gomez 1997) and high-resolution images (Wood et al. 1998) of T Tauri stars has favored a grain mixture which Kim, Martin, & Hendry (1994, hereafter KMH) derived from detailed fits to the interstellar extinction law. Important grain properties are the opacity κ, the scattering albedo ω, and the scattering asymmetry parameter g. The latter defines the forward throwing properties of the dust and ranges from 0 (isotropic scattering) to 1. What sets the KMH grains apart is that they are more forward throwing ($g = 0.40[R]$, $0.25[K]$), and have higher albedo ($\omega = 0.50[R]$, $0.36[K]$) at each wavelength (optical to near-IR). They are also less polarized, but that is a property we do not use here. The grain size distribution has the lower cutoff of KMH (0.005 μm) and a smooth exponential falloff, instead of an upper cutoff, at 0.25 μm. Since the dust settling time is proportional to size^{-1}, we performed calculations with upper cutoffs of 0.05 and 0.1 μm. None of these had any significant effect on the temperatures.

4. THE TEMPERATURE STRUCTURE AND THE SNOW LINE

We are interested in planet formation and therefore want to find the ice condensation line ("snow line") in the mid-

plane of the disk. Temperature inversions in the disk's vertical structure (see D'Alessio et al. 1998) may lead to lower temperatures above the midplane, but ice condensation there is quickly destroyed upon crossing the warmer disk plane. We define the snow line simply in terms of the local gas-dust temperature in the midplane, and at a value of 170 K.

In our passive disk, under hydrostatic and radiative equilibrium, the vertical and radial temperature profiles are similar to those of Chiang & Goldreich (1997) and $T(r) \propto r^{-3/7}$. Here is why: the disk has a concave upper surface (see Kenyon & Hartmann 1987) with pressure scale height of the gas at the midplane temperature, h:

$$\frac{h}{r} = \left(\frac{rkT}{GM_*\,\mu m_{\rm H}}\right)^{1/2},$$

where G is the gravitational constant, μ and $m_{\rm H}$ are the molecular weight and hydrogen mass, r is radius in the disk, and T is the midplane temperature at that radius. For the inner region (but $r \gg R_*$) of a disk with such concave shape, the stellar incident flux $F_{\rm irr}(r) \propto \phi(r)\sigma T_*^4 r^{-2}$, where $\phi(r) \propto r^{2/7}$ (see next section). Here T_* is the effective temperature of the central star. Then our calculation makes use of the balance between heating by irradiation and radiative cooling: $\sigma T^4(r) = F_{\rm irr}(r)$. Therefore our midplane temperature will scale as $T(r) = T_0 r^{-3/7}$ K. This is not surprising, given our standard treatment of the vertical hydrostatic structure of the disk irradiated at angles $\phi(r)$. Only the scaling coefficient, $T_0 = 140$, will be different. The difference with the Chiang & Goldreich model is our treatment of the dust grains—less energy is redistributed inward in our calculation and the midplane temperature is lower (Fig. 1). The model with accretion heating is much warmer inward of 2.5 AU where it joins the no-accretion (passive) model—stellar irradiation dominates.

The result above is for our model with $M_* = 0.5\,M_\odot$ and $T_* = 4000$ K, which is standard for T Tauri stars. It is interesting to see how the snow line changes for other realistic initial parameters. By retaining the same dust properties, this can be achieved using scaling relations rather than
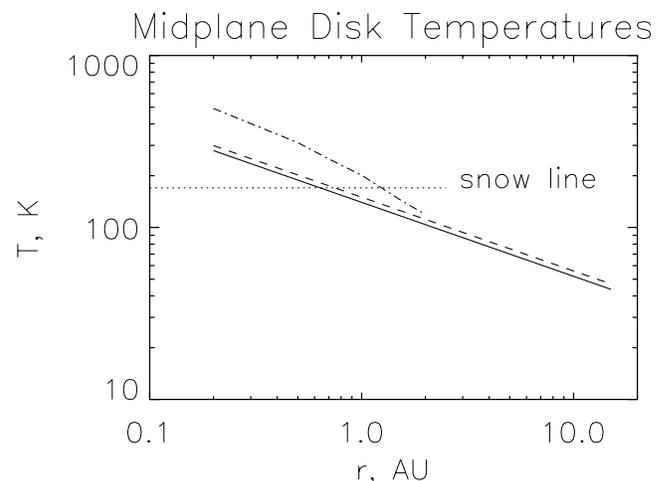

FIG. 1.—Temperature profiles in the midplane for the two models computed: passive (*solid line*) and with accretion heating (*dash-dotted line*). The passive disk model of Chiang & Goldreich (1997) is shown also (*dashed line*).

complete individual models as shown by Bell et al. (1997) for the pre–main-sequence mass range of 0.5–2 $M_\odot$. An important assumption at this point is that we have still retained the same (minimum-mass solar nebula) disk. With our set of equations, the midplane temperature coefficient, T_0, will be proportional to the stellar mass: $T_0 \propto M_*^{3/(10-k)}$, where k is a function of the total opacity in the disk and $0 \le k < 2$.

On the other hand, different disk masses for a fixed central star ($M_* = 0.5\ M_\odot$ and $T_* = 4000$ K) can be modeled for zero accretion rate, by changing Σ_0 by a factor of 10 in each direction (defined in § 2). Here a remaining assumption is the radial dependence of $\Sigma \propto r^{-3/2}$; the latter could certainly be $\propto r^{-1}$ (Cameron 1995), or a more complex function of r, but it is beyond the intent of our paper to deal with this. Moreover we find minimal change in the midplane temperature in the r range of interest to us (0.1–5 AU). The reason is a near cancellation that occurs between the amount of heating and increased optical depth to the midplane. One could visualize the vertical structure of a passive disk for $r = 0.1$–5 AU as consisting of three zones: (1) optically thin heating and cooling region (dust heated by direct starlight); (2) optically thin cooling, but optically thick heating layer; and (3) the midplane zone, where both heating and cooling occur in optically thick conditions. The rate of stellar heating of the disk per unit volume is directly proportional to the density and affects the location and temperature of the irradiation layer. That is nearly canceled (except for second-order terms) in the mean intensity which reaches the midplane. Therefore we find that $T(r)$ changes within ± 10 K for a change in disk density (Σ_0) of a factor of 10. Note that $T(r)$ is only approximately $\propto r^{-3/7}$, even for $r = 0.1$–5.0 AU; the small effect of density on $T(r)$ has an r dependence. However, for the purposes of this paper, i.e., our chosen volume of parameter space, the effect of disk mass on $T(r)$ and the snow line is insignificant, and we do not pursue the issue in more detail. Note that for a disk with a heat source in the midplane, i.e., with an accretion rate different from zero, the midplane $T(r)$ will be strongly coupled to the density, roughly $\propto \rho^{1/4}$, and will increase at every r for higher disk masses (e.g., Lin & Papaloizou 1985).

5. THE SHAPE OF THE UPPER SURFACE OF THE DISK

The "snow line" calculation in the previous section is made under the assumption that the upper surface of the disk is perfectly concave and smooth at all radii, r. This is a very good description of such unperturbed disks because thermal and gravitational instabilities are damped very efficiently (D'Alessio et al. 1999). Obviously this is not going to be the case when an already formed planet core distorts the disk. But even a small distortion of the disk's surface may affect the thermal balance. The distortion need only be large enough compared to the grazing angle at which the starlight strikes the disk, $\phi(r)$:

$$\phi(r) = \frac{0.4 R_*}{r} + r \frac{d}{dr}\left(\frac{h}{r}\right),$$

where h is the local scale height. This small angle has a minimum at 0.4 AU and increases significantly only at very large distances: $\phi(r) \approx 0.005 r^{-1} + 0.05 r^{2/7}$ (e.g., see Chiang & Goldreich 1997).

The amount of compression due to the additional mass of the planet, M_p, will depend on the Hill radius, $R_H = r(M_p/M_*)^{1/3}$, and how it compares to the local scale height, h. The depth of the depression will be proportional to $(R_H/h)^3$. The resulting depressions (one on each side) will be in the shadow from the central star, with a shade area dependent on the grazing angle, $\phi(r)$. The solid angle subtended by this shade area from the midplane determines the amount of cooling and the new temperature in the sphere of influence of the planet core. The question then arises of whether during the timescale preceding the opening of the gap, the midplane temperature in the vicinity of the accreting planet core could drop below the ice condensation limit even for orbits with r much shorter than the "snow line" radius in the undisturbed disk. The answer appears to be affirmative, and a runaway develops whereby local ice condensation leads to rapid growth of the initial rocky core, which in turn deepens the depression in the disk and facilitates more ice condensation inside the planet's sphere of influence. Details about the instability which develops in this case will be given in a separate paper.

6. CONCLUSION

When the large fraction of close-in extrasolar giant planets became apparent, we thought of questioning the standard notion of a distant "snow line" beyond 3 AU in a protoplanetary disk. Thence comes this paper. We revisited the issue by paying attention to the stellar irradiation and its radiative effects on the disk, thus limiting ourselves to passive or low accretion rate disks.

We find a snow line as close as 0.7 AU in a passive disk and not much farther away than 1.3 AU in a disk with 10^{-8} $M_\odot$ yr^{-1} accretion rate for $M_* = 0.5\ M_\odot$. The result is robust regardless of different reasonable model assumptions—similar values could, in principle, be inferred from existing disk models (Chiang & Goldreich 1997; D'Alessio et al. 1998). For more massive (and luminous) central stars, the snow line shifts outward to 1.0 AU (1 $M_\odot$) and 1.6 AU (2 $M_\odot$). The effect of different disk mass is much smaller for passive disks—the snow line shifts inward by 0.08 AU for $\Sigma_0 = 10^4$ g cm^{-2}. Our results differ from existing calculations (in that they bring the snow line even closer in) because the dust grains properties we used have higher albedo and are more forward throwing. The dust grains and the disk models we used are typical of T Tauri stars of age ~ 1 Myr. So our conclusion is that if such T Tauri disks are typical of protoplanetary disks, then the snow line in them could be as close in as 0.7 AU.

Our estimate of the snow line is accurate to within 10%, once the model assumptions are made. These assumptions are by no means good or obvious and can change the numbers considerably. For a passive disk model, the assumptions that need to be justified are the equilibrium of the disk, the lack of dust settling (i.e., gas and dust are well mixed), the used KMH properties of the dust grains, and the choice of molecular opacities. For a low accreting disk model, one has to add to the above list the choice of viscous dissipation model (and $\alpha = 0.01$).

Finally, we note that these estimates reflect a steady state disk in hydrostatic equilibrium. The disk will get disturbed as planet formation commences, which may affect the thermal balance locally given the small value of the grazing angle, ϕ. For a certain planet core mass, an instability can

develop at orbits smaller than 1 AU, which can lead to the formation of giant planets in situ. What is then the determining factor for the division between terrestrial and giant planets in our solar system remains unexplained (as it did even with a snow line at 2.7 AU).

REFERENCES

Bell, K. R., Cassen, P. M., Klahr, H. H., & Henning, Th. 1997, ApJ, 486, 372
Boss, A. P. 1996, ApJ, 469, 906
———. 1998, Annu. Rev. Earth Planet. Sci., 26, 53
Calvet, N., Patino, A., Magris, G., & D'Alessio, P. 1991, ApJ, 380, 617
Cameron, A. G. W., 1995, Meteoritics, 30, 133
Chiang, E. I., & Goldreich, P. 1997, ApJ, 490, 368
D'Alessio, P., Canto, J., Calvet, N., & Lizano, S. 1998, ApJ, 500, 411
D'Alessio, P., Canto, J., Hartmann, L., Calvet, N., & Lizano, S. 1999, ApJ, 511, 896
Draine, B. T., & Lee, H. M. 1984, ApJ, 285, 89
Hartmann, L., Calvet, N., Gullbring, E., & D'Alessio, P. 1998, ApJ, 495, 385
Hayashi, C. 1981, Prog. Theor. Phys. Suppl., 70, 35

Ivezic, A., & Elitzur, M. 1997, MNRAS, 287, 799
Kenyon, S. J., & Hartmann, L. 1987, ApJ, 323, 714
Kim, S. H., Martin, P. G., & Hendry, R. D. 1994, ApJ, 422, 164 (KMH)
Kurucz, R. 1992, in IAU Symp. 149, Stellar Population of Galaxies, ed. B. Barbuy & A. Renzini (Dordrecht: Kluwer), 225
Lin, D. N. C., & Papaloizou, J. 1985, in Protostars and Planets II, ed. D. C. Black & M. S. Matthews (Tucson: Univ. Arizona Press), 981
Mathis, J. S., Rumpl, W., & Nordsieck, K. H. 1977, ApJ, 217, 425
Menshchikov, A. B., & Henning, Th. 1997, A&A, 318, 879
Pringle, J. E., 1981, ARA&A, 19, 137
Whitney, B. A., Kenyon, S. J., & Gomez, M. 1997, ApJ, 485, 703
Wood, K., Kenyon, S. J., Whitney, B. A., & Turnbull, M. 1998, ApJ, 497, 404