DYNAMICAL MODELS OF WINDS FROM ROTATING HOT STARS

by

Steven Robert Cranmer

A dissertation submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics

Fall 1996

© 1996 Steven Robert Cranmer
All Rights Reserved
DYNAMICAL MODELS OF WINDS FROM ROTATING HOT STARS

by

Steven Robert Cranmer

Approved:

Henry R. Glyde, Ph.D.
Chairman of the Department of Physics and Astronomy

Approved:

John C. Cavanaugh, Ph.D.
Interim Associate Provost for Graduate Studies
I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: 
Stanley P. Owocki, Ph.D.
Professor in charge of dissertation

I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: 
James MacDonald, Ph.D.
Member of dissertation committee

I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: 
Derck Massa, Ph.D.
Member of dissertation committee

I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: 
David Seckel, Ph.D.
Member of dissertation committee

I certify that I have read this dissertation and that in my opinion it meets the academic and professional standard required by the University as a dissertation for the degree of Doctor of Philosophy.

Signed: 
Gary Zank, Ph.D.
Member of dissertation committee
ACKNOWLEDGEMENTS

First and foremost I would like to express my gratitude to my advisor and friend, Dr. Stanley Owocki, for essential guidance and insight throughout my graduate career. More than anyone else, his influence has contributed to my development as a scientist.

Sincere appreciation is due to Dr. Alex Fullerton and Dr. Kenneth Gayley, who have enlarged my sphere of understanding of astrophysics with their fresh perspectives. I also wish to thank Dr. James MacDonald, Dr. Derck Massa, Dr. David Seckel, and Dr. Gary Zank for taking the time to serve on my thesis committee and also for many useful conversations. I am also indebted to Dr. George Collins and Dr. Dermott Mullan for their helpful advice, original ideas, and encyclopaedic knowledge.

Additionally, I thank everyone at the Bartol Research Institute for the unique and stimulating environment they have provided, and most notably: Nick Arge, Dr. Steve Barr, Sujit Basu, Dave Huber, Dave McKenzie, Dr. Louis Pauls, Dr. Todd Story, and Dr. Lance Williams. I am especially grateful to Dr. Norman Ness, the NASA Space Grant Committee, and the NASA Graduate Student Researchers Program for making my work at Bartol possible.

Needless to say, my family deserves a great deal of credit for my development. I thank my mother Patricia and my father Roy for all their love and support. Finally, I offer my most genuine thanks to my wife Janet, whose unwavering compassion, faith, and love carries me through life.
# TABLE OF CONTENTS

LIST OF FIGURES ........................................... ix  
LIST OF TABLES ........................................... xiii

Chapter

1 INTRODUCTION ........................................... 1

1.1 Motivation for Hot Star Wind Research ................... 1  
1.2 Overview of the Observations ............................... 3  
  1.2.1 The Existence of Stellar Winds .......................... 3  
  1.2.2 The Effects of Rotation ................................. 5  
  1.2.3 Variability and Inhomogeneity .......................... 7  
1.3 Goals of this Work ...................................... 10

2 THE THEORY OF RADIATIVELY DRIVEN STELLAR WINDS ... 12

2.1 Equations of Radiation Hydrodynamics .................... 12  
2.2 The Sobolev Radiation Force ............................... 17  
  2.2.1 The Continuum and Individual Lines .................... 17  
  2.2.2 Simple Force Estimates ................................. 21  
  2.2.3 General Line Ensemble Forces .......................... 25  
  2.2.4 The Finite Stellar Disk ................................. 27  
  2.2.5 The Effect of the Ionization Balance .................. 30  
2.3 Wind Solutions .......................................... 32  
  2.3.1 Nonlinear Solution Methods ............................ 32
6 DYNAMICAL MODELS OF COROTATING WIND STRUCTURE .............................................. 128

6.1 The Local Radiative Force Enhancement ............................................. 129
6.2 Numerical Results ................................................................. 134

6.2.1 Standard Bright Spot: Model 1 .............................................. 135
6.2.2 Standard Dark Spot: Model 2 ............................................... 143
6.2.3 Variation of Spot Amplitude, Width, and Stellar Rotation Velocity ............................................. 146

6.3 Synthetic Observational Diagnostics ............................................. 153

6.3.1 SEI Line Profile Construction ................................................ 153
6.3.2 Time Variability in Dynamical Models .................................... 155
6.3.3 Continuum Polarization Variations ......................................... 161

6.4 Summary, Conclusions, and Future Work ....................................... 166

7 PULSATIONS, WAVES, AND DISCONTINUITIES IN STELLAR WINDS ................................. 169

7.1 Global Stellar Pulsation ............................................................ 170

7.1.1 Linearized Hydrodynamic Equations ...................................... 170
7.1.2 Simple Oscillatory Solutions ............................................... 172
7.1.3 Surface Constraints ............................................................ 175
7.1.4 Discrete Frequency Eigenvalues .......................................... 177
7.1.5 The Effects of Rotation ...................................................... 180

7.2 Wave Propagation in Winds ....................................................... 184

7.2.1 Basic Hydrodynamic Equations ............................................. 184
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2.2</td>
<td>Linearization</td>
<td>185</td>
</tr>
<tr>
<td>7.2.3</td>
<td>Local Dispersion Analysis</td>
<td>190</td>
</tr>
<tr>
<td>7.2.4</td>
<td>A Homogeneous Medium</td>
<td>192</td>
</tr>
<tr>
<td>7.2.5</td>
<td>A Stratified Atmosphere</td>
<td>196</td>
</tr>
<tr>
<td>7.2.6</td>
<td>A Subsonic Wind</td>
<td>201</td>
</tr>
<tr>
<td>7.2.7</td>
<td>A Supersonic Wind</td>
<td>202</td>
</tr>
<tr>
<td>7.2.8</td>
<td>Large-Scale Wave Propagation Analysis</td>
<td>205</td>
</tr>
<tr>
<td>7.3</td>
<td>Nonlinear Wave Effects</td>
<td>209</td>
</tr>
<tr>
<td>7.3.1</td>
<td>Weak-Shock Wave Steepening</td>
<td>209</td>
</tr>
<tr>
<td>7.3.2</td>
<td>Discontinuities in Fluid Variables</td>
<td>212</td>
</tr>
<tr>
<td>7.3.3</td>
<td>Discontinuities in Gradients</td>
<td>214</td>
</tr>
<tr>
<td>8</td>
<td>SUMMARY AND CONCLUSIONS</td>
<td>217</td>
</tr>
<tr>
<td>8.1</td>
<td>Overview of Current Work</td>
<td>217</td>
</tr>
<tr>
<td>8.2</td>
<td>Future Research Goals</td>
<td>220</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
<td>223</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

2.1 Spherical-star finite disk (FD) factors, plotted for both a uniformly bright star and a linearly limb-darkened star. .......................... 31

2.2 Solution topology for the mCAK wind model of ζ Puppis. .......... 43

2.3 Parameter study for various mCAK wind models of ζ Puppis. .. 45

3.1 Theoretical P Cygni line profiles for the standard ζ Puppis model, varying the dimensionless line strength. ............................. 61

3.2 Theoretical P Cygni line profiles for the standard ζ Puppis model, varying the microturbulent wind velocity. .......................... 62

3.3 Theoretical P Cygni line profiles for a strong saturated line, varying the microturbulent wind velocity. ............................... 63

3.4 Theoretical P Cygni line profiles for the standard ζ Puppis model, varying the scaled probability of collisional deexcitations. .... 64

4.1 Equatorial cross sections of Roche equipotential surfaces, plotted for a uniform distribution of ω values between 0 and 1.05, at increments of 0.05. The dotted curve is a representative post-breakup surface (ω = 1.05). ................................. 71

4.2 Equatorial “Koninx effect” velocity laws for rotating mCAK models of ζ Puppis. ................................................................. 79

4.3 Relative mass flux variation with colatitude θ and rotational angular velocity ω, for the pure centrifugal enhancement of Section 4.2.1 and the von Zeipel gravity darkening modulation of Section 4.2.2. 83
4.4 Comparison of strong and weak horizontal plane-parallel radiative acceleration, and the existence of steep shear solutions. 88

4.5 Vertical velocity laws and dimensionless steep shear solutions for a plane-parallel rotating wind. 91

5.1 Coordinate geometry for the computation of the oblate finite disk (OFD) factor. The star-centered (un-primed) and wind-centered (primed) coordinate systems are shown, related by the position of the field point. 100

5.2 Contour and line plots of the radial continuum OFD factor for a rapidly rotating B2 star. 107

5.3 Contour and line plots of the latitudinal continuum OFD factor for a rapidly rotating B2 star. 108

5.4 Contour and line plots of the radial line OFD factor for a rapidly rotating B2 star. 110

5.5 Contour and line plots of the latitudinal line OFD factor for a rapidly rotating B2 star. 111

5.6 Contour and line plots of the azimuthal line OFD factor for a rapidly rotating B2 star. 112

5.7 Theoretical (SEI) P Cygni line profiles for the A1 analytic wind compression model wind. Observers at $i = 0, 22.5, 45, 67.5, and 90$ degrees are shown. 116

5.8 Optically-thin electron scattering polarization for the A1 analytic wind compression model and the S-350 numerical hydrodynamics wind model. 118

5.9 Mass-loading source terms $j(r)$ and $J(r)$ for the 1D idealized wind compressed disk model. 122

5.10 Solution topology to the 1D wind compressed disk equation of motion, without radiation or centrifugal forces. 126
6.1 Contours of the CIR star-spot force enhancement for a spot with full width at half maximum $\Phi = 20$ degrees. 132

6.2 Variation with spot amplitude (for $\Phi = 20$ degrees) of (a) radial velocity and (b) density at a radius of 10 stellar radii, for non-rotating winds. 133

6.3 CIR structure for Model 1, settled to a steady state. Shown are the (a) density, (b) radial velocity, (c) azimuthal velocity, and (d) radial Sobolev optical depth, all normalized to the unperturbed wind initial condition. 137

6.4 Streamlines and streaklines for Model 1, integrated from 72 equally-spaced points on the star, separated by 5 degree intervals. 139

6.5 Normalized density gray-scale for Model 1, overplotted with various physical features of the flow. 142

6.6 Line plots for Model 1 of the radial variation of (a) radial velocity and (b) density in the equatorial plane at 16 equally-spaced azimuthal angles, 11.25 degrees apart. 144

6.7 As in Figure 6.3, except for Model 2. 145

6.8 As in Figure 6.6, except for Model 2. 147

6.9 Velocity minima and maxima corresponding to CIR compression and radiative-acoustic kink features, shown for Model 1 and variable amplitude Models 3A, 3B, 3C. 148

6.10 As in Figure 6.9, except for Model 1 and variable spot-width Models 4A, 4B, 4C. 149

6.11 As in Figure 6.9, except for Model 1 and variable rotation Models 5A, 5B. 150

6.12 Radial velocity showing the radiative-acoustic kink and CIR shock for models with varying spot amplitude. 152

6.13 SEI absorption-column line variability for Model 1, computed for an unsaturated line. 156
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.14</td>
<td>As in Figure 6.13, except for a strong saturated line.</td>
</tr>
<tr>
<td>6.15</td>
<td>As in Figure 6.13, except for Model 2.</td>
</tr>
<tr>
<td>6.16</td>
<td>Best-fit DAC features: central line velocity, characteristic width, and slab-model column depth, shown for Model 1 and variable amplitude Models 3A, 3B, 3C.</td>
</tr>
<tr>
<td>6.17</td>
<td>As in Figure 6.16, except for Model 1 and variable spot-width Models 4A, 4B, 4C.</td>
</tr>
<tr>
<td>6.18</td>
<td>As in Figure 6.16, except for Model 1 and variable rotation Models 5A, 5B.</td>
</tr>
<tr>
<td>6.19</td>
<td>DAC acceleration versus velocity for Model 1 (dashed line) and analytic “beta” velocity laws (solid lines).</td>
</tr>
<tr>
<td>6.20</td>
<td>The upper plot shows the magnitude of polarization $P$ varying with azimuthal phase, for observers at $i = 0, 22.5, 45, 67.5, 90$ degrees. The lower plot shows the individual $Q$ and $U$ variations over one full cycle, or $180$ degrees of phase.</td>
</tr>
<tr>
<td>7.1</td>
<td>Discrete NRP eigenperiods for an idealized B supergiant model. The gray hatched regions denote periods and horizontal wavenumbers which can propagate radially in an isothermal photosphere.</td>
</tr>
<tr>
<td>7.2</td>
<td>Fractional density amplitudes for near-star wind oscillations of a B supergiant. Solid lines represent modes that are able to propagate in the static isothermal photosphere, and dotted lines represent evanescent modes.</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

2.1 Parameter study for various mCAK wind models of ζ Puppis . . 46

5.1 Oblate finite disk (OFD) wind models . . . . . . . . . . . 103

5.2 Oblate finite disk (OFD) wind models, continued . . . . . . 104

6.1 Summary of CIR parameter study . . . . . . . . . . . . . 136
ABSTRACT

The hottest and most massive stars (spectral types O, B, Wolf-Rayet) have strong stellar winds that are believed to be driven by line scattering of the star’s continuum radiation field. The atmospheres and winds of many hot stars exhibit the effects of rapid rotation, pulsation, and possibly surface magnetic fields, inferred from observations of ultraviolet spectral lines and polarization. The complex time variability in these observations is not yet well understood. The purpose of this dissertation is to model the dynamics of winds around rotating hot stars and synthesize theoretical observational diagnostics to compare with actual data.

Before dealing with rotation, however, we derive the theory of radiative driving of stellar winds, and uncover several new useful aspects of the theory for spherical, nonrotating stars. The presence of limb darkening of the stellar radiation is found to be able to increase the mass flux \( \dot{M} \) by 10–15\% over standard models assuming a uniformly-bright star, and the wind’s asymptotic terminal velocity \( v_\infty \) should decrease by the same amount. We also introduce a new approximation method for estimating the terminal velocity, which is both conceptually simpler and more physically transparent than existing approximation algorithms. Finally, from theoretical line profile modeling we find that observational determinations of \( v_\infty \) may be underestimated by several hundred km s\(^{-1}\) if unsaturated P Cygni lines are used.

Rotation affects a star by introducing centrifugal and Coriolis forces, decreasing the effective gravity and making the star oblate. This in turn redistributes the emerging radiative flux to preferentially heat the stellar poles, an effect known as gravity darkening. Although previous models have computed the increase in equatorial mass flux due to the lower effective gravity there, none have incorporated gravity darkening. We find that the brighter (darker) flux from the poles (equator) has a much stronger impact on the mass flux, increasing (decreasing) the mass loss and local wind density. This, in addition to the existence of nonradial radiation forces from a rotating star, which tend to point latitudinally away from the equator and azimuthally opposite the rotation, produces a net poleward deflection of wind streamlines. This is contrary to the “wind compressed disk” model of Bjorkman
and Cassinelli, and also seems incompatible with observational inferences of equatorial density enhancements in some systems. This work is ongoing, and we are endeavoring to include all the relevant physics in hydrodynamical simulations.

We also dynamically model spectral-line time variability by inducing corotating nonaxisymmetric structure in the equatorial plane of a hot-star wind. By varying the radiation force over localized “star spots,” the wind develops fast and slow streams which collide to form corotating interaction regions (CIRs) similar to those in the solar wind. We synthesize P Cygni type line profiles for a stationary observer, and find that “discrete absorption components” (DACs) accelerate slowly through the profiles as complex nonlinear structures rotate in front of the star. We also examine the photospheric origin of such variability, in a preliminary manner, by deriving the theory of stellar pulsations, waves, and discontinuities. Although most observed low-order pulsation modes are evanescently damped in the photosphere, we find that the presence of an accelerating wind can allow waves of all frequencies to propagate radially. We thus make a first attempt at outlining the possible “photospheric connection” between interior and wind variability that observations are beginning to confirm.