AN INTERPRETATION OF CYCLE PERIODS OF STELLAR CHROMOSPHERIC ACTIVITY

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ABSTRACT

We propose \( (P_{\text{cye}}/P_{\text{rot}})^2 \), the square of the ratio of the characteristic oscillatory timescales of stellar chromospheric activity to the rotation period, as a useful parameterization of the stellar activity cycle and as the observational equivalent of the theoretical dynamo number, \( N_D \). \( (P_{\text{cye}}/P_{\text{rot}})^2 \) can be obtained observationally from 25 yr activity records of stellar Ca II H and K chromospheric emission fluxes of the Mount Wilson Observatory HK Project.

Using that parameterization, we study the relationships between the period of the activity cycle and mass or age (estimated from the average level of chromospheric emission, \( \langle R_{\text{HK}} \rangle \), and its calibration with age). The quantity \( (P_{\text{cye}}/P_{\text{rot}})^2 \) increases as \( B-V \) decreases, in qualitative agreement with the expectation that as the fractional depth of convective zone decreases (i.e., toward higher mass stars), \( N_D \) increases (i.e., the variability of stellar activity tends to be more irregular). \( (P_{\text{cye}}/P_{\text{rot}})^2 \) seems independent on age for young stars but has a well-defined dependence on age for the older stars. The difference in the behavior of \( (P_{\text{cye}}/P_{\text{rot}})^2 \) with age is another aspect of chromospheric activity that changes as a star ages; the time of the transition depends on mass but occurs roughly near stellar age of ~1–3 billion years.

Subject headings: stars: activity — stars: chromospheres — stars: late-type — stars: magnetic fields — stars: rotation

1. INTRODUCTION

The Ca II H (396.8 nm) and K (393.3 nm) chromospheric emission fluxes in ~100 lower main-sequence stars have been continually monitored at Mount Wilson Observatory since 1966. The 25 yr records of Ca II H and K emission fluxes relative to the nearby photospheric fluxes (the S-index) trace the variations of surface magnetic field (e.g., Baliunas et al. 1993). \( S \) is the ratio of fluxes in 1 Å passbands centered on H and K to fluxes in 20 Å passbands centered at 390.1 nm and 400.1 nm. The net chromospheric emission can be represented by the parameter \( R_{\text{HK}} \), which is corrected for the photospheric contribution in the H and K passbands of the S-index and is normalized by the total bolometric luminosity (Middlekauff 1982; Noyes et al. 1984a). \( R_{\text{HK}} \) allows a comparison of the activity levels in stars of different masses.

Three important quantities can be derived from the stellar activity records: (1) the time-averaged level of the chromospheric emission, \( \langle R_{\text{HK}} \rangle \) (averaged over the 25 yr interval of observations); (2) the stellar rotation, \( P_{\text{rot}} \) (Baliunas et al. 1983, 1993); and (3) the activity cycle period, \( P_{\text{cye}} \). The mean level of chromospheric emission, \( \langle R_{\text{HK}} \rangle \), is associated with the average level of surface magnetic flux (i.e., the magnetic field strength and the magnetic field area coverage—the filling factor) and depends on a star's age (Soderblom, Duncan, & Johnson 1991). The quantity, \( P_{\text{rot}} \), changes as a star loses angular momentum over evolutionary timescales (hence, it, too, is associated with the age of the star, e.g., Kraft 1967). However, comparing \( P_{\text{rot}} \) in stars of different masses is nontrivial (see below). The third quantity, \( P_{\text{cye}} \), measures the average timescale associated with the generation and regeneration of magnetic fields and can be identified with the characteristic turbulent magnetic diffusion timescale, \( \tau_p \), from stellar dynamo theory (see below). Approximately one-third of the stars that were monitored display cyclic activity behavior with periods ranging from 3 to 21 yr (Wilson 1978; Baliunas et al. 1993).

The mean level of chromospheric emission is inversely related to the rotation period (Noyes et al. 1984b and Rutten 1987) but changes slope at a particular \( P_{\text{rot}} \) or Rossby number (\( R_o = P_{\text{rot}}/\tau_c \), where \( \tau_c \) is the convective over-turn time; e.g., Durney & Latour 1978). The activity-rotation relation might be interpreted in terms of stellar dynamo operation. However, the activity-rotation fit is quite sensitive to the values of the ratio of mixing length to scale height used (Gilliland 1985). We suggest that the interpretation in terms of a dynamo would be more straightforward if based on the \( (P_{\text{cye}}/P_{\text{rot}})^2 \) parameterization discussed below.

The quantity, \( (P_{\text{cye}}/P_{\text{rot}})^2 \), seems well suited for discussing the dependence of the stellar activity cycle on mass and age. Furthermore, as suggested below, \( (P_{\text{cye}}/P_{\text{rot}})^2 \) may be interpreted as the observational equivalent of the stellar dynamo number, \( N_D \) (Parker 1979). This choice is guided by the observations and attempts to avoid the uncertainties associated with model-dependent normalization parameters, e.g., \( \tau_D \) for \( P_{\text{cye}} \) (Tuominen, Rudiger, & Brandenburg 1988) and \( \tau_c \) for \( P_{\text{rot}} \) (Noyes et al. 1984a).

2. RESULTS AND DISCUSSION

The stellar rotation, \( P_{\text{rot}} \), and the dominant period of cyclic activity, \( P_{\text{cye}} \), are determined from the chromospheric activity records by a periodicogram analysis suited for an unevenly spaced data set (Horne & Baliunas 1986); details of the analysis are discussed in Baliunas et al. (1993). The significance of a period can be estimated by the false alarm probability (FAP). We assume a period to be significant if FAP \( \leq 10^{-2} \).

The rotation periods have been determined by analyzing individual seasonal time series; the activity cycle periods are obtained from the full interval of observations. All of the rotation periods detected range within ±25% (or less) of the mean and the mean value of \( P_{\text{rot}} \) is adopted here. (For some stars the...
spread in $P_{\text{rot}}$ may include differential surface rotation; Donahue & Baliunas 1992.) The values of $P_{\text{cyc}}$ are determined from records covering 1–3 activity cycles, and have a precision of roughly $\pm 1$ year (Baliunas et al. 1993). However, the sunspot record since 1750 reveals an "intrinsic" range of 9–13 yr in the values of $P_{\text{cyc}}$. If the values of stellar $P_{\text{cyc}}$ also have a physical range, similar to the solar behavior, then it is difficult to assess formally the uncertainties of $P_{\text{cyc}}$.

Five of the stars have two significant periods detected which are not related as through an alias. The physical origin of multiple periodicities is not well understood (e.g., Hoyng 1990), although the recent proposal of Boyer & Levy (1992) of a multilayer dynamo with varying magnetic diffusivity in each layer seems capable of producing a multiple-mode dynamo. However, their work may not be directly applicable to the double periodicities observed here since their investigation focused on periodicities which involved larger relative difference in timescales (e.g., the 22 yr and the 50 to 130 yr periods in the sunspot record).

2.1. Dependence of $(P_{\text{cyc}}/P_{\text{rot}})^2$ on Mass and Age

In Figure 1, we study the dependence of $(P_{\text{cyc}}/P_{\text{rot}})^2$ on the $B-V$ index (i.e., mass for lower main-sequence stars). Stars are distinguished by their ranges of $\langle R_{\text{HK}} \rangle$ and $P_{\text{rot}}$: one group has high levels of average activity and fast rotation; the other group has lower levels of activity and slower rotation. (The groupings are based on criteria similar to those of Noyes et al. 1984.) We show linear least-squares fits based on smaller sets of data which have very well determined cycle periods ($FAP \leq 10^{-5}$), in order to help identify the underlying trend of the relation for the larger sample ($FAP \leq 10^{-4}$). Assuming that $(P_{\text{cyc}}/P_{\text{rot}})^2$ is associated with the dynamo model (see below), we find that the general trend in Figure 1 indicates that $N_{\text{ch}}$ decreases as $B-V$ increases.

In Figure 2, we study the dependence of $(P_{\text{cyc}}/P_{\text{rot}})^2$ on stellar age (using $\langle R_{\text{HK}} \rangle$) for all stars ($FAP \leq 10^{-4}$). We find that the $(P_{\text{cyc}}/P_{\text{rot}})^2$–age relation for the older group is statistically significant with $r = 0.748$, $n = 16$, and a probability of null hypothesis of 0.001 (excluding the upper point of the "double-period" star HD 100180). A two-dimensional Kolmogorov-Smirnov test for the existence of two dissimilar stellar groups (distinguished by the activity level and rotation criteria mentioned above) yields a K-S measure, $D \approx 0.8$, which invalidates the null hypothesis that the two sets of data are drawn from the same distribution function. (The significance level represents the fraction of Monte Carlo simulations of synthetic values of $D$ that exceed the observed value of $D$). However, a statistical significance can be misleading unless supported by some physical arguments. We will attempt (below) to provide interpretations of the two distinct groups in terms of the operation of stellar dynamos.

We have estimated stellar age from $\langle R_{\text{HK}} \rangle$ using the empirical power-law fit given by Soderblom et al. (1991). The power-law fit is used instead of the exponential fit (or the fit for constant star formation rate which is an exponential plus a power-law function) to preserve the presentation of the results in Figure 2 as a function of age or $\langle R_{\text{HK}} \rangle$. We note that an accurate estimate of age is not available and that the correct functional form of the chromospheric emission-age relation is still unresolved. We also note that applying the exponential fit may introduce an artificial enhancement of the observed "break" between the $(P_{\text{cyc}}/P_{\text{rot}})^2$ trends between the young and older stellar groups.

The distinct difference in stellar dynamos operating at fast or slow rotation rates (hence age) for a given mass has been interpreted by Knobloch, Rosner, & Weiss (1981) as the transition from convection in rolls parallel to the rotation axis to normal convection cells as the angular velocity of a star decreases. However, direct numerical verification for this suggestion is not available. Another relevant theoretical work is that of

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**Fig. 1**—The ratio $(P_{\text{cyc}}/P_{\text{rot}})^2$ is plotted as a function of $(B-V)$ for all activity cycle periods detected (false alarm probability, $FAP \leq 10^{-5}$). The filled circles denote stars with high levels of activity and fast rotation (hence young); the symbol $\bigcirc$ denotes the Sun and the open circles denote stars with lower level of activity and slower rotation (hence old). The two lines are the least squares fits for a smaller set of data ($FAP \leq 10^{-5}$); i.e., stars which have very well defined cycle periods; data points for HD 156026 and 201092 were excluded from the fit to indicate the possible underlying trends in the $(P_{\text{cyc}}/P_{\text{rot}})^2(B-V)$ relation. Dashed vertical lines connect the two independent periods detected in five of the "double-period" stars.

**Fig. 2**—The ratio $(P_{\text{cyc}}/P_{\text{rot}})^2$ is plotted as a function of age estimated from average level of chromospheric emission $\langle R_{\text{HK}} \rangle$–age relation (power-law fit) of Soderblom et al. (1991). The filled symbols denote stars with high level of activity and fast rotation (hence young); the symbol $\bigcirc$ denotes the Sun and the open symbols denote stars with lower level of activity and slower rotation (hence old). The line for the older stellar group is the least-squares fit using all the data (except for the upper point of HD 100180). The line for the younger stellar group was fitted using a smaller set of data with $FAP \leq 10^{-5}$ to indicate the possible underlying trend. Dashed vertical lines connect the two independent periods detected in five of the "double-period" stars. Two sets of $B-V$ ranges (triangles are stars with $0.55 \leq B-V \leq 0.66$ and squares are stars with $0.75 \leq B-V \leq 0.86$) highlight the $(P_{\text{cyc}}/P_{\text{rot}})^2$–age dependence for two limited ranges of mass.
2.2. Association of \( (P_{\text{cy}}/P_{\text{rot}})^2 \) with Stellar Dynamo Number, \( N_D \)

The two basic ingredients for generation and regeneration of the magnetic fields are that of the large-scale differential rotation, \( \Delta \Omega \), and the small-scale turbulent transport in the convective zone due to the nonzero helicity, \( \alpha \) (i.e., nonzero \( \mathbf{v} \cdot \mathbf{V} \times \mathbf{v} \)). Since the observed chromospheric emission activities are surface phenomena, we avoid discussion on the possibility of the complex dependence of \( \alpha \) and \( \Delta \Omega \) on stellar internal structure. We have assumed that the dynamo operates near the base of the convective zone (e.g., DeLuca & Gilman 1991).

From mean field dynamo theory, the dynamo number, \( N_D \), is given by

\[
N_D = \frac{\alpha \Delta \Omega d^3}{\eta^2},
\]

(1)

where \( d \) is the characteristic length scale for the convection and \( \eta \) is the turbulent magnetic diffusivity. We have also explicitly assumed that the dynamo number is above its critical value (because \( P_{\text{cy}} \) is detected) so that periodic solutions of the nonlinear dynamo equation exist (e.g., Weiss et al. 1984). The following approximate expressions for \( \alpha \) and \( \Delta \Omega \) are adopted (Durney & Robinson 1982),

\[
\alpha = c_1 \frac{d^2 \Omega}{R_c} f_1(|B|),
\]

(2)

\[
\Delta \Omega = c_2 \Omega \left( \frac{d}{R_c} \right)^2 f_2(|B|),
\]

(3)

where \( c_1, c_2 \) are adjustable constants, \( R_c \) is the inner radius of the convective zone, \( \Omega \) is the average angular velocity (= 2\( \pi/P_{\text{rot}} \)) and \( f_1(|B|), f_2(|B|) \) are the possible nonlinear feedbacks of the magnetic field on helical turbulent transport and differential rotation, respectively (\( f_1 = f_2 \equiv 1 \) in linear theory).

In addition, the diffusivity, \( \eta \), is modified in the presence of magnetic field, \( \eta = \eta_0 f_3(|B|) \) where \( \eta_0 \) is the molecular value of the diffusivity (e.g., Cattaneo & Vainshtein 1991).

Substituting for \( \alpha, \Delta \Omega, \eta \) in equation (1), we get

\[
N_D = c_1 c_2 \frac{f_1(|B|) f_2(|B|) \left( \frac{d}{R_c} \right)^3 \left( 2\pi \tau_B \right)}{f_3(|B|)^2 \left( \frac{P_{\text{cy}}}{P_{\text{rot}}} \right)^2}.
\]

(4)

Thus, the dynamo number, \( N_D \), is directly proportional to the square of the ratio between the diffusion time scale, \( \tau_B = \eta_0 d^2/\eta \), and the stellar rotation period, \( P_{\text{rot}} \). It is clear that a complete justification for the association of \( (P_{\text{cy}}/P_{\text{rot}})^2 \) with \( N_D \) will be difficult (for example, the possibility of broken spatial symmetries in the structure of mathematical equations that are used to model magnetic field generation in stellar dynamos can lead to further complexities; Jennings & Weiss 1991).

However, we propose to adopt the association to allow for a much-needed diagnostic of the information on the key parameters, such as \( \alpha, \Delta \Omega, \) and \( \eta \) in equation (1). Later, the association can be reexamined by studying nonlinear dynamo models with the inferred parameters. Although this process is circular, it provides a close working relation between the observational and theoretical understanding of the stellar dynamo.

Noyes, Weiss, & Vaughan (1984b) previously suggested that dynamo action is limited for the less active stars (those with \( P_{\text{cy}} \gtrsim 20 \) days, roughly equivalent to the older stellar group of this work) primarily by losses due to magnetic buoyancy. In a parametric study of various nonlinear dynamo models, they used the observed power-law relation between \( \log(P_{\text{cy}}) \) and \( \log(P_{\text{rot}}) \) [or \( \log(R_o) \)] for the less-active stars as a constraint that ruled out quenching of the \( \alpha \)-effect or of differential rotation. Although we have adopted a slightly different approach (i.e., we consider that the exact parameterization of the stellar activity cycle is difficult to obtain for stars of different mass and age), we can nevertheless perform a similar study based on the results of the dynamo models studied by Noyes et al. (1984a). We studied the \( \log(P_{\text{cy}}) \) versus \( \log(R_o) \) relation (where \( \tau_B \) was calculated using the formula provided by Noyes et al. 1984a) with the updated stellar data set (with \( \text{FAP} < 10^{-5} \)).

For the young stellar group, our analysis suggests dynamo models favoring an equal combination of quenching of the \( \alpha \)-effect, differential rotation, and losses through magnetic buoyancy. For the older stars, the observations indicate dynamo models that are consistent with limitation by the fluctuations in differential rotation generated by the Lorentz force (e.g., Yoshimura 1981), a mechanism which also seems to simulate the torsional oscillations observed on the Sun (Howard & LaBonte 1980). The difference in our interpretation and that of Noyes et al. (1984b) for the older stars suggests that the analysis is sensitive to the qualitative values of the slope in \( \log(P_{\text{cy}}) \) versus \( \log(R_o) \) [or \( \log(P_{\text{rot}}) \)] relation and that perhaps the identification of the different mechanisms that limit the dynamo action is premature.

3. SUMMARY

Although the observations of Ca II H and K emission are confined to surface phenomena, the information that they provide may be important for understanding the stellar dynamo. Our analysis of the relatively small sample of stars has relied directly on observed characteristics of the variation of surface magnetic flux. The correlation between \( (P_{\text{cy}}/P_{\text{rot}})^2 \) and age for the older stellar group was found to be statistically significant, which suggests a common mechanism governing their surface activities, although the specific physical process remains elusive. We also find the distribution of \( (P_{\text{cy}}/P_{\text{rot}})^2 \) as a function of age for the young and old stellar groups (Fig. 2) to be statistically different. If the association of \( (P_{\text{cy}}/P_{\text{rot}})^2 \) with \( N_D \) can be confirmed, the results in Figure 2 may serve as direct evidence for the existence of a break in the stellar dynamo efficiency as a star ages, over the range of age studied. The results obtained from \( (P_{\text{cy}}/P_{\text{rot}})^2 \)-parameterization may also provide a useful diagnostic of details of nonlinear feedbacks of magnetic field on the fluid motions in stellar convective zones.

A break in dynamo efficiency as a star ages would explain the gap in the chromospheric activity in stars within 25 pc of the Sun (Vaughan & Preston 1980). Such interpretation (see also Durney, Mihalas, & Robinson 1981) differs from Hartmann et al. (1984), who interpreted the break in the stellar activity as a fluctuation in the local stellar birthrate, based on
the assumption of a smoothly varying relation of chromospheric emission with age (although they did not rule out the possibility of "fine-scale discontinuities or rapid transitions in chromospheric behavior").

Even with some indication that the long-term behavior of stellar magnetic activity (including the Maunder-Minimum-like magnetically flat intervals [Eddy 1976; Baliunas & Jastrow 1990]) can be described within the framework of deterministic chaos (e.g., Weiss et al. 1984; Schmalz & Stix 1991), the physical mechanisms responsible for different behavior of the underlying stellar dynamo remain difficult to identify.

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REFERENCES

Eddy, J. A. 1976, Science, 192, 1189

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